National Center for Computational Hydroscience and Engineering
The University of Mississippi

Verification and Validation of Numerical models and their Applications in Hydraulic Engineering

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Earlier Soil Conservation Problems in the US

Headcut erosion: one of the sediment sources

House partially buried by sediment

Water is not always Beautiful !

Introduction

- **Needs for numerical model verification and validation**
- **Principles of verification and validation**
- **V&V for CCHE3D/2D free surface flow models**
- **Applications in hydrodynamic, sediment transport and morphologic processes**

Serious Consequences

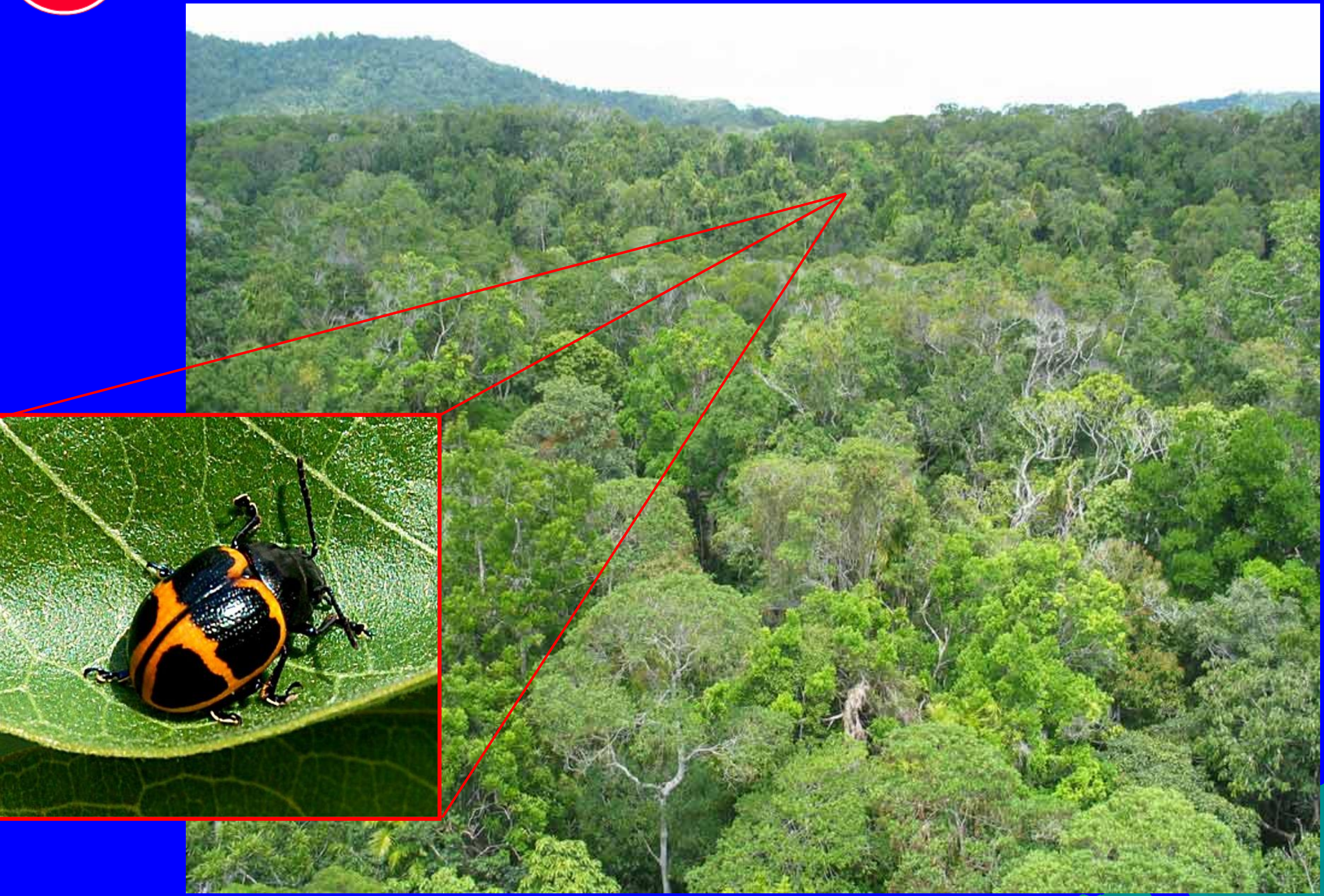
- *NASA's Mariner 1 was destroyed due to code error (July 22,1962).*
- Just 293 seconds after launch, a range safety officer ordered a destructive abort when it veered off course after an unscheduled yaw-lift maneuver.
- *Verification: "a missing hyphen in coded computer instruction in the data-editing program allowed transmission of incorrect guidance signals".*
- ***The cost of Mariners 1 through 10 was*** *approximately $135 million, making that missing hyphen an expensive mistake.*

Other serious problems

- *Sleipner A, offshore oil rig had a catastrophic failure in the North Sea (August 23,1991).*
- *The failure resulted from an error caused by un-conservative concrete codes and inaccurate finite element analysis modelling in the design of the structure.*
- *Financial loss was estimated $180M to $700M*

Computer-Aided Catastrophes

How to find a bug in a jungle?

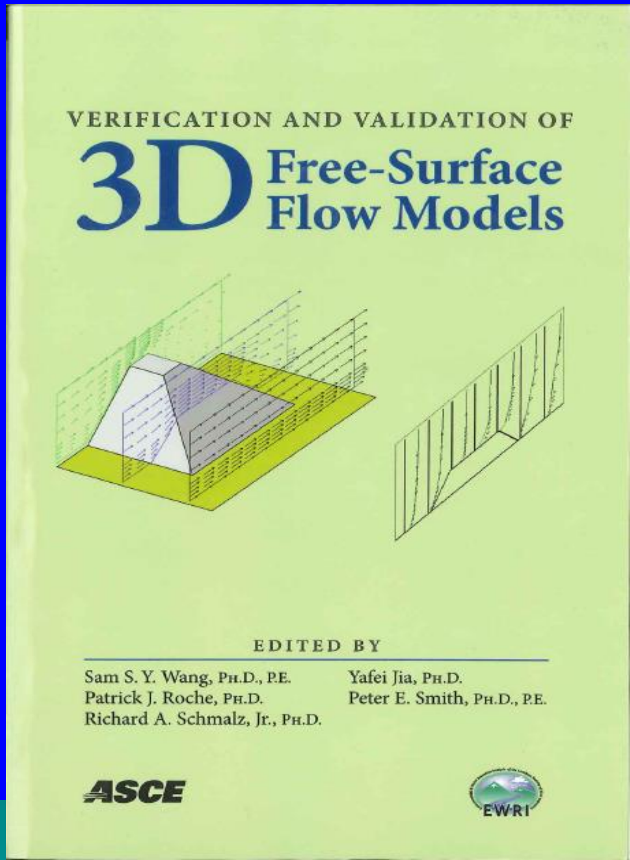


NUMERICAL MODEL VERIFICATION & VALIDATION

- **Mathematical Verification**
  Mathematical (derivation, solution, programming) errors
  Convergence and Quantitative Error

- **Physical Validation**
  Capable of reproducing basic physical processes

- **Site Specific Field Validation**
  Calibration of Model Parameters
  Validation of Over-All Accuracy



Verification:

Solve the equation right

Validation:

Solve the right equation

I. Mathematical Verification

Prescribed Solution Forcing or Manufactured Solution Method

Rationale:
The best way to verify a numerical model is to compare its solution to analytical solutions of the differential equations. It is, however, very difficult to obtain non-trivial solutions, MSM suggests to use manufactured arbitrary solutions for model verification

For a differential equation $A(U)=0$ (1)

A manufactured solution V is an arbitrary analytic function of space and time.

Insert V into Eq.1, One has

$$A(V)=f \,, \quad f\neq 0 \qquad (2)$$

f is a known analytic function obtained simply by calculus derivation

V now is a known analytic solution of Eq. 2.

Include analytic form of f in the numerical model as source terms

The deferential equation (2) can be used to solve V numerically

Manufactured Solution I

Three dimensional, unsteady and non-linear solutions are "manufactured" for numerical verification

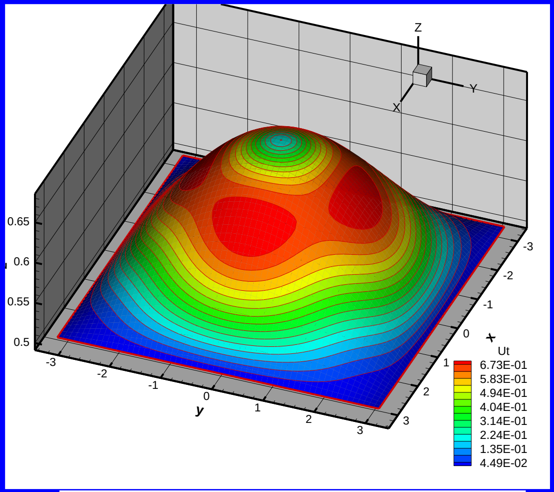
$$u = \sin y \cos^2\frac{x}{2}\cos\frac{\pi}{2}(1-\frac{z}{h})\sin t$$

$$v = -\sin x \cos^2\frac{y}{2}\cos\frac{\pi}{2}(1-\frac{z}{h})\sin t$$

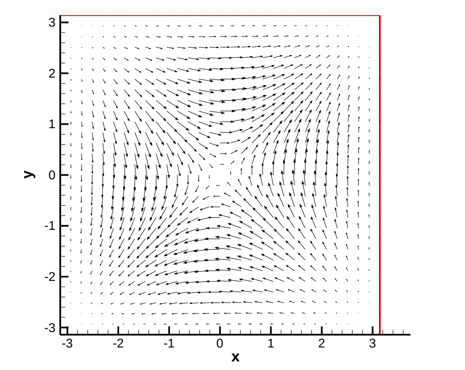
$$w = -A\cos\frac{x}{2}\cos\frac{y}{2}\cos\frac{\pi}{2}(1-\frac{z}{h})\sin t$$

$$h = A\cos\frac{x}{2}\cos\frac{y}{2}\cos t + h_0$$

$$p = C_p \cos(x)\cos(y)\cos(\frac{\pi}{2}\frac{z}{h})$$



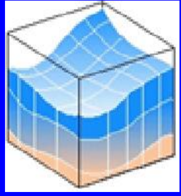
The 3D view of the surface shape and velocity magnitude distribution on the surface



Velocity vector field at $z=0.5h$

Note h satisfies the free surface kinetic BC

$$\frac{\partial h}{\partial t} + u_h\frac{\partial h}{\partial x} + v_h\frac{\partial h}{\partial y} - w_h = 0$$



Manufactured Solution II

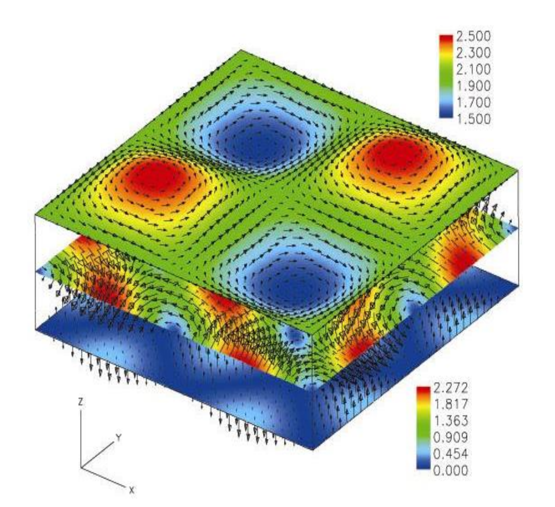
Steady state 3D none-linear manufactured solution with a free surface

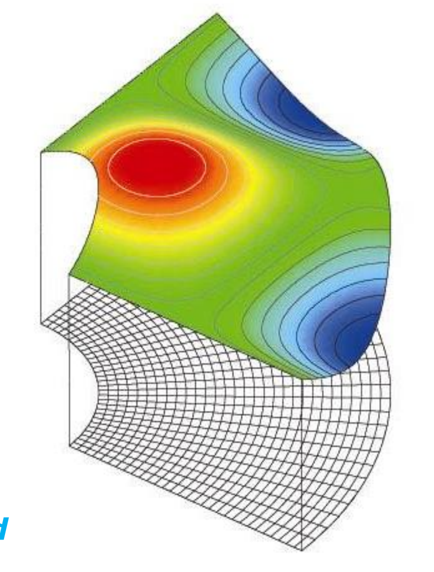
$$u = \sin(x)\cos(y)\sin(\frac{\pi}{2}\frac{z}{h}) - \cos(x)\sin(y)[\cos(2\pi\frac{z}{h}) - 1]$$

$$v = -\cos(x)\sin(y)\sin(\frac{\pi}{2}\frac{z}{h}) + \sin(x)\cos(y)[\cos(2\pi\frac{z}{h}) - 1]$$

$$w = -\frac{A}{2\pi}[\sin^2(x)\cos^2(y) - \cos^2(x)\sin^2(y)][\sin(2\pi\frac{z}{h}) - 2\pi\frac{z}{h}\cos(2\pi\frac{z}{h}) + 2\pi]$$

$$h = A\sin(x)\sin(y) + h_0$$





Note *u*, *v*, *w* satisfies mass conservation $\frac{\partial u_i}{\partial x_i} = 0$

Note *h* satisfies the free surface kinetic BC $\frac{\partial h}{\partial t} + u_h\frac{\partial h}{\partial x} + v_h\frac{\partial h}{\partial y} - w_h = 0$

Governing Equations of 3D Free Surface Flows

Momentum conservation:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + f_i + \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}[\nu_t(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})] = 0$$

Mass conservation:

$$\frac{\partial u_i}{\partial x_i} = 0$$

Free Surface kinetic:

$$\frac{\partial h}{\partial t} + u_h \frac{\partial h}{\partial x} + v_h \frac{\partial h}{\partial y} - w_h = 0$$

v_t=constant

Inserting Solution I into the momentum equation, one has to calculate

$$f_u = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - \nu_t(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) - \nu_t(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y \partial x} + \frac{\partial^2 w}{\partial z \partial x}) - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

For example: analytic form of source term for $u\frac{\partial u}{\partial x}$

$$u = \sin y \cos^2 \frac{x}{2} \cos\frac{\pi}{2}(1 - \frac{z}{h}) \sin t$$

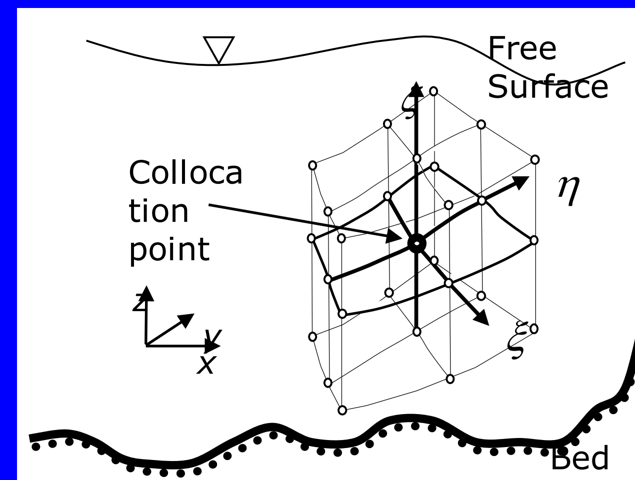
$$\frac{\partial u}{\partial x} = -\frac{1}{2}\sin x \sin y \cdot C \cdot \sin t + \sin y \cos^2 \frac{x}{2}\frac{\partial C}{\partial x}\sin t$$

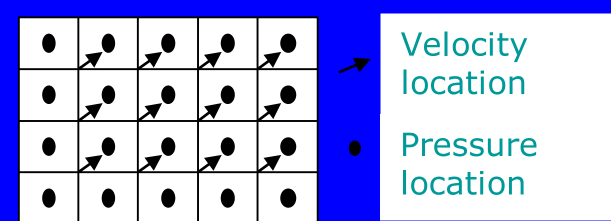
$$C = \cos[\frac{\pi}{2}(1 - \frac{z}{h})]$$

CCHE3D verified using *MSM*

Efficient element method combining finite element and finite volume approach

- Non-uniform quadrilateral grid
- collocation approach
- Partially staggered pressure grid
- Hydro-static/dynamic pressure
- Free surface
- Non-oscillation
- Wet/dry moving boundary
- Modulated coding method





$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}[\nu_t(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})] = f_i$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

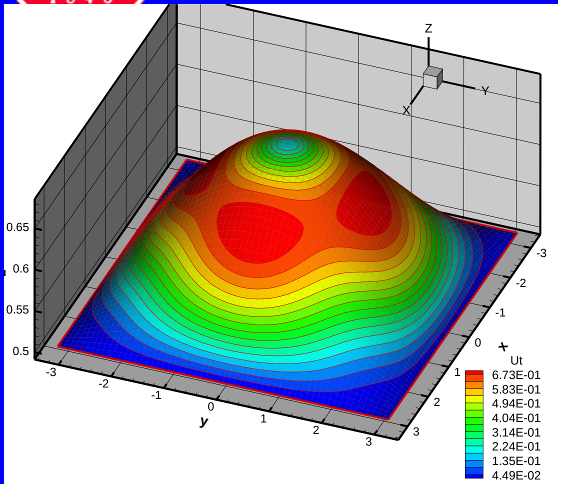
$$\frac{\partial h}{\partial t} + u_h \frac{\partial h}{\partial x} + v_h \frac{\partial h}{\partial y} - w_h = 0$$

Error estimation and convergence

$$Err_u = \sqrt{\frac{\sum (u_a - u_n)^2}{(I_{\max} - 2)(J_{\max} - 2)(K_{\max} - 2)}}$$

$$E = f(\Delta) - f_a = C\Delta^p + H.O.T.$$

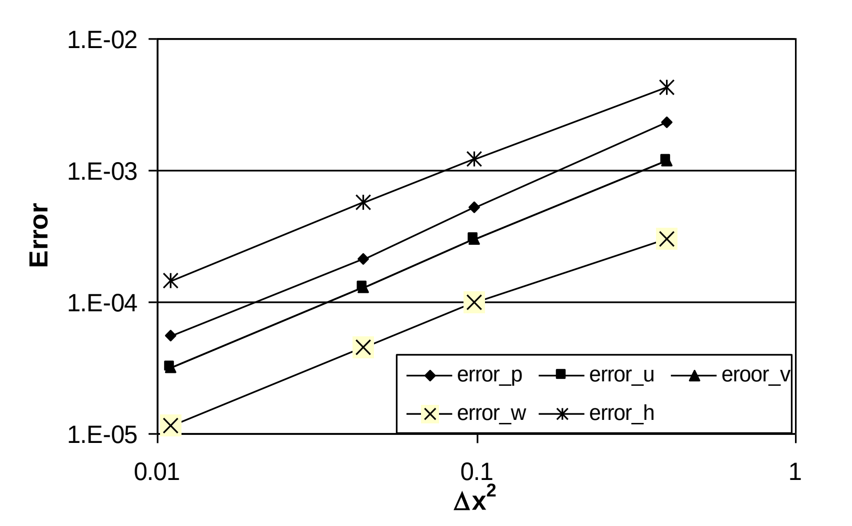
Verification using Solution I



$$E = C\Delta^2$$

Parameters	A=0.5m, $t=t_0$ $\Delta t=0.01\pi$		A=0.5m, $t=t_0$ $\Delta t=0.001\pi$	
	C	R^2	C	R^2
Error-u	0.0068	0.9999	0.003	1.
Error-v	0.0068	0.9999	0.003	1.
Error-w	0.0035	0.9996	0.0007	0.9937
Error-p	0.0055	0.9995	0.0059	0.9995
Error-h	0.0437	0.9999	0.0107	0.9989

Convergence for a steady state solution ($t=t_0$)



The time t in the source term and boundary conditions are set to be t_o All boundary conditions are of Dirichlet except at water surface: $\frac{\partial u_i}{\partial x_3} = 0$

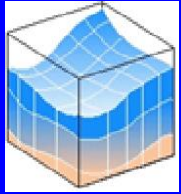
F	Linear terms only	1st order upwinding		Convective interpolation 1.6 order upwinding		2nd order upwinding		QUICK scheme			
	A=0.5	*A*=0.5		*A*=0.5		*A*=0.5		*A*=0.5		*A*=0.0	
	C	*C*	R^2	*C*	R^2	*C*	R^2	*C*	R^2	*C*	R^2
Error-u	0.0017	0.0041	0.9999	0.0021	0.9998	0.0021	1.0	0.0017	1.0	0.0011	1.0
Error-v	0.0017	0.0041	0.9999	0.0021	0.9998	0.0021	1.0	0.0017	1.0	0.0011	1.0
Error-w	0.0002	0.0008	1.0	0.0003	0.9999	0.0002	0.9996	0.0002	0.9999	0.00005	0.999

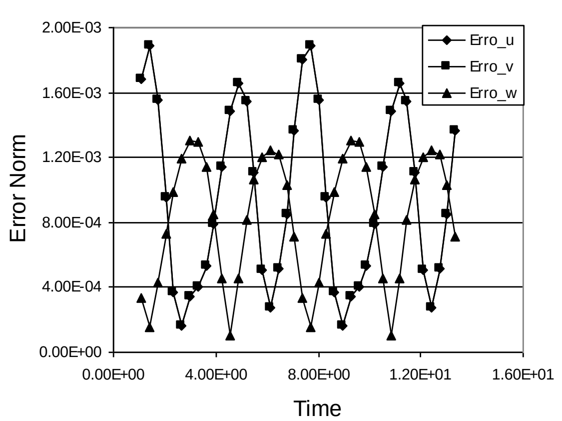
$E = C\Delta^2$ $E = C\Delta^{1.0}$ $E = C\Delta^{1.6}$ $E = C\Delta^2$ $E = C\Delta^2$

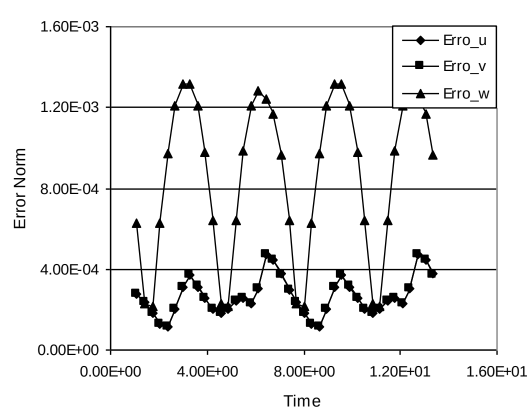
Test cases regard to non-linear terms using *Function I*

- Value of *C* indicates error level, the exponent indicates convergence; R^2 indicates consistency
- Linear terms alone shows the lowest errors and 2nd order convergence
- Error will increase when advection terms are included
- First order upwinding shows the highest error
- Quick scheme is 2nd order and shows the lowest error among all schemes tested
- Error is smaller without mesh distortion (*A*=0)
- Convergence behavior of each term could be tested individually

Convergence of unsteady cases





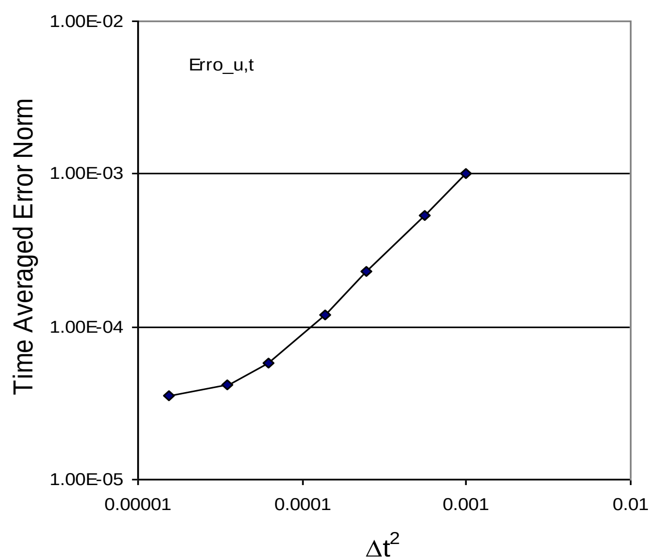


Error norm using first order Euler and QUICK scheme

When unsteady cases are considered, error norms vary in time.
Averaged error norms are used to evaluate convergence due to time step size

$$Err_{u_i,t} = \frac{1}{N_t}\sum_n Err_{u_i}$$

Error norm using second order Euler and QUICK scheme

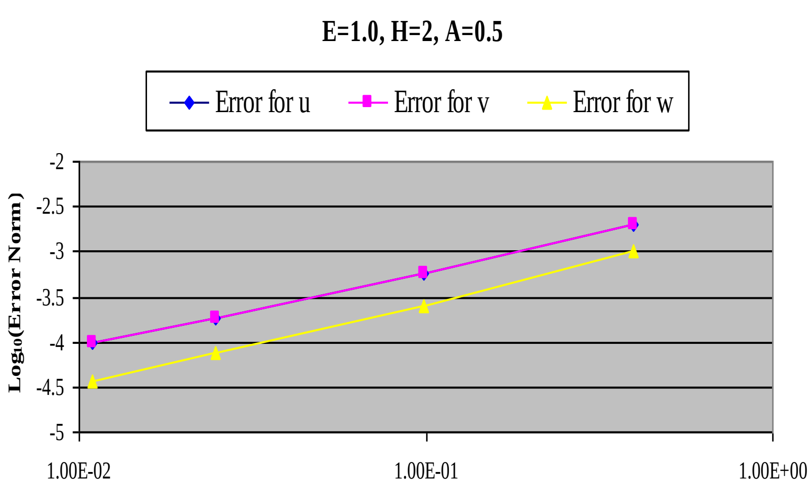


- Time averaged error norm using a second order corrected Euler time marching scheme
- Time step is varied with fixed mesh
- Second order convergence is achieved larger *Δt*.
- When time step is small, the errors due to time and space are getting close in magnitude, the convergence trend flattened

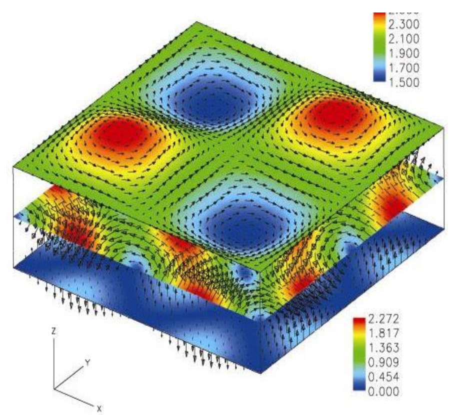
Δt	0.00125π	0.001875π	0.0025π	0.00375π	0.005π	0.0075π	0.01π

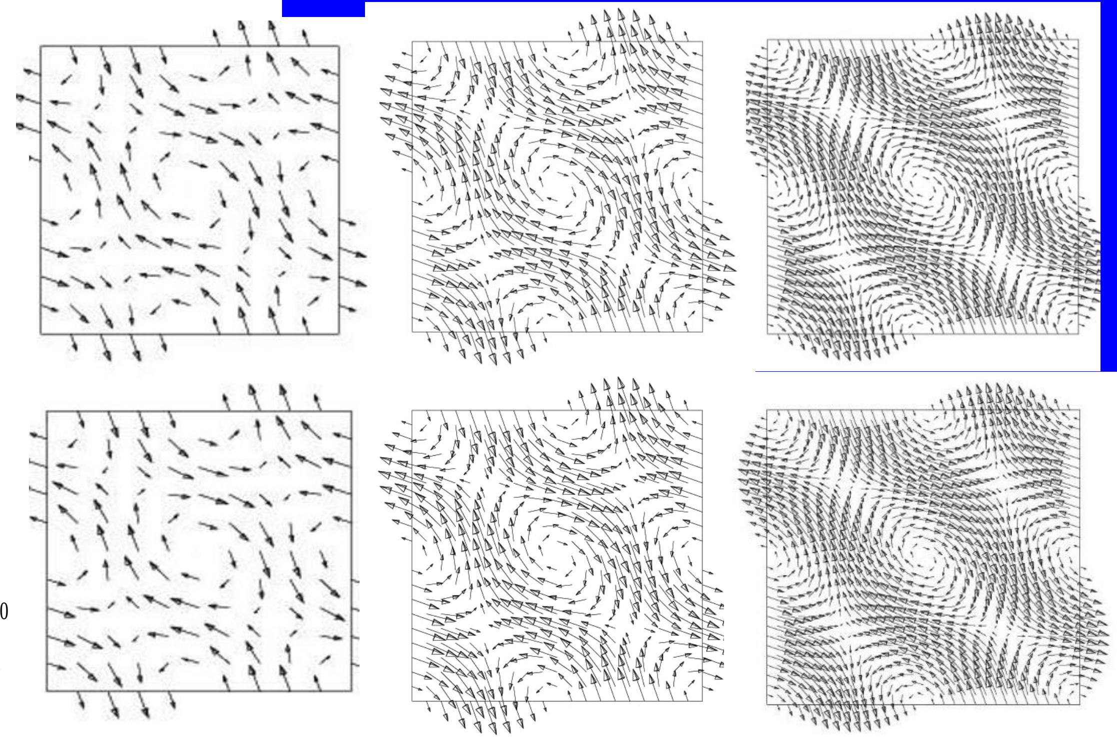
Verification using Solution II

Manufactured solution



Numerical solution





Comparisons in the middle level of the domain: z/h=0.5

Mesh: 11x11x11	Mesh: 21x21x21	Mesh: 61x61x61

$Err_u = c_u \Delta x^2$	$c_u = 0.0015$	$R^2 = 0.9936$
$Err_v = c_v \Delta x^2$	$c_v = 0.0015$	$R^2 = 0.9930$
$Err_w = c_w \Delta x^2$	$c_w = 0.0012$	$R^2 = 0.995$

Findings

1. One bug was identified and corrected which reduce the dynamic pressure accuracy in deformed element from 2^{nd} order to 1^{st} order.

2. Identified the accuracy of upwinding schemes:

   Convective interpolation: 1.6 order

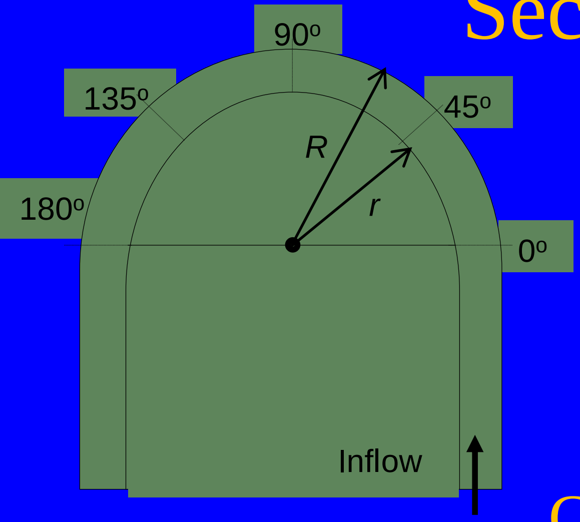
   Quick scheme : 2.0 order with small error coefficient

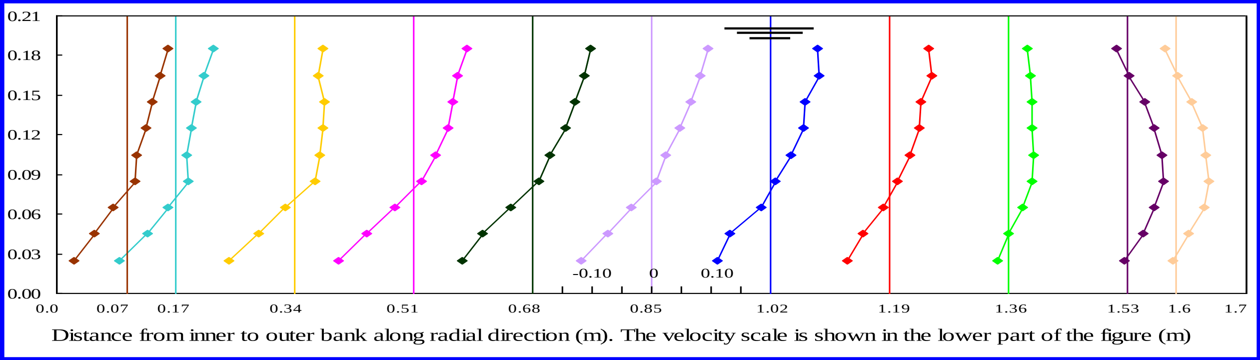
3. The MSM is effective to identify derivation/coding errors. But it has to be done in the developer level.

Numerical model validation

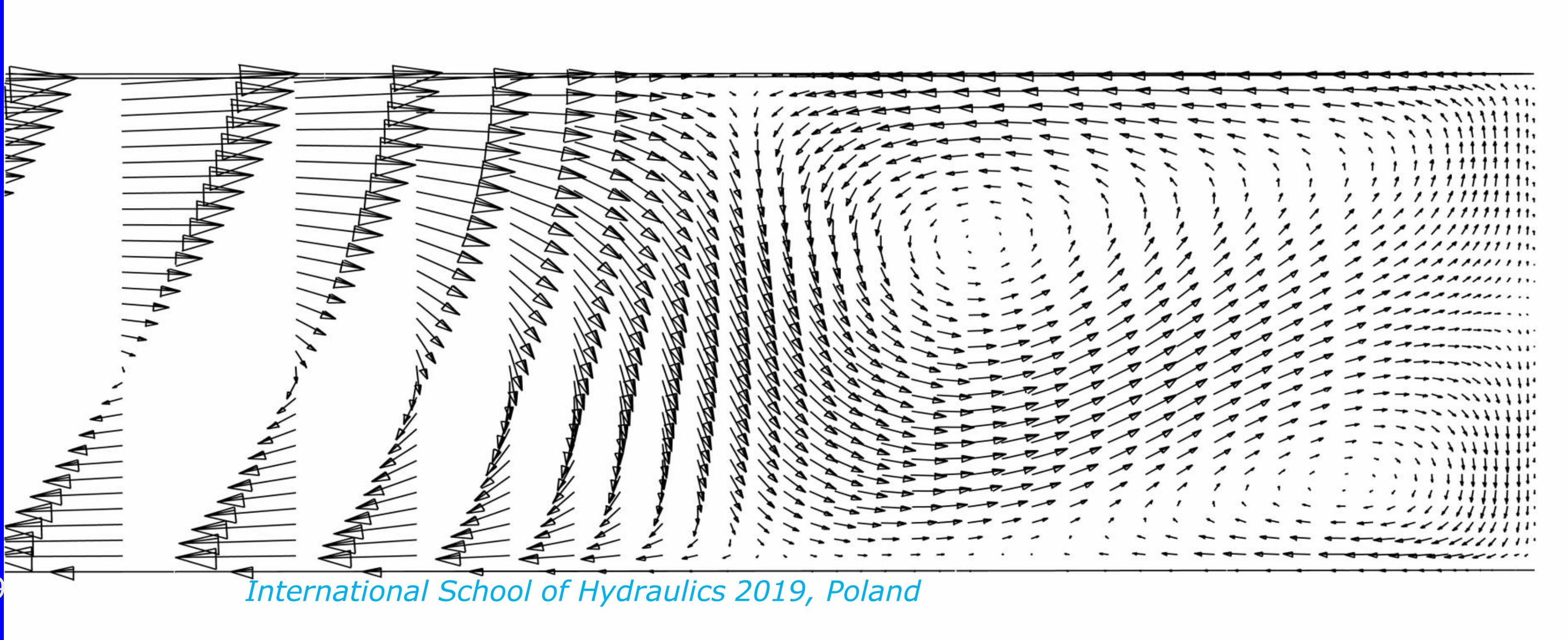
Examples validating CCHE3D/2D using physical experimental data

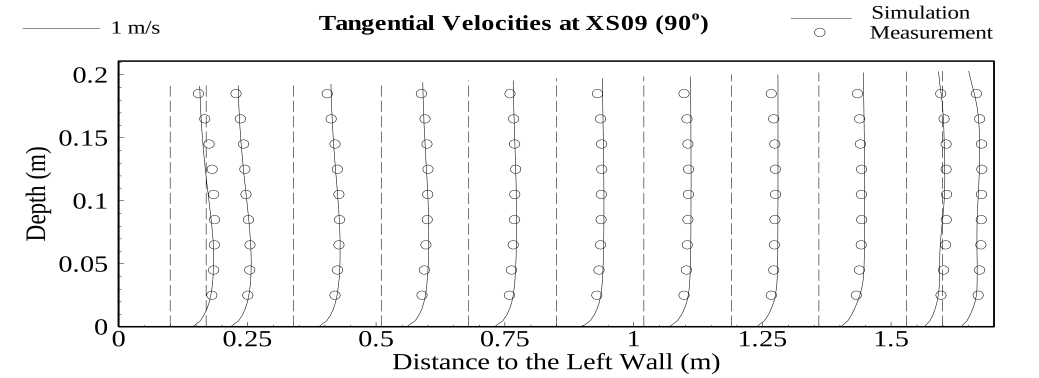
Secondary currents in 90º cross-section

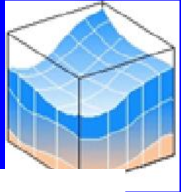


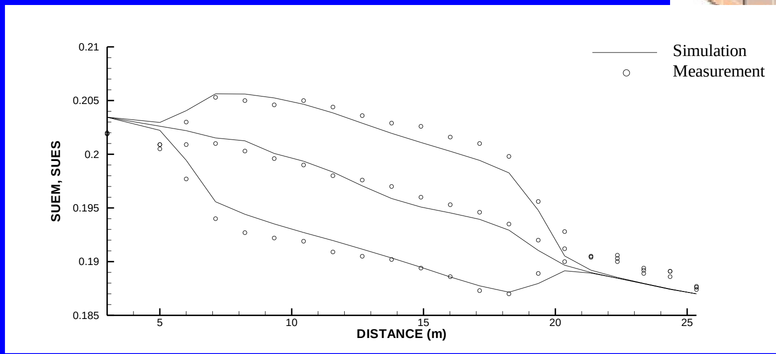


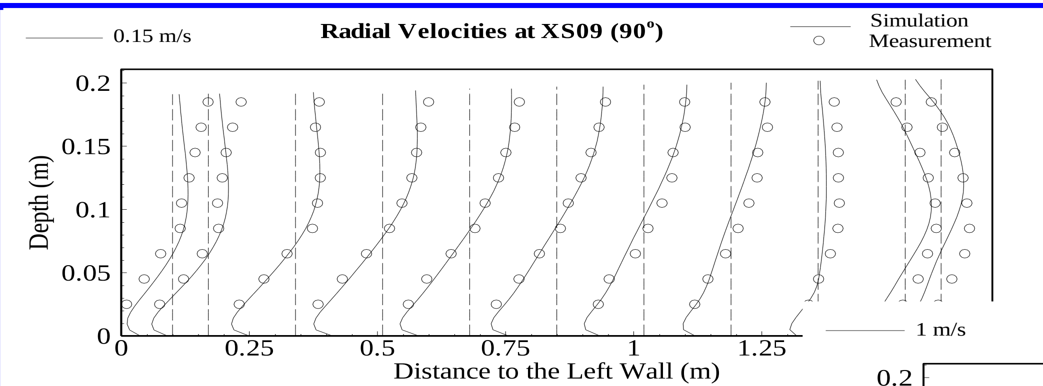
CCHE3D model with non-linear k-ε closure model

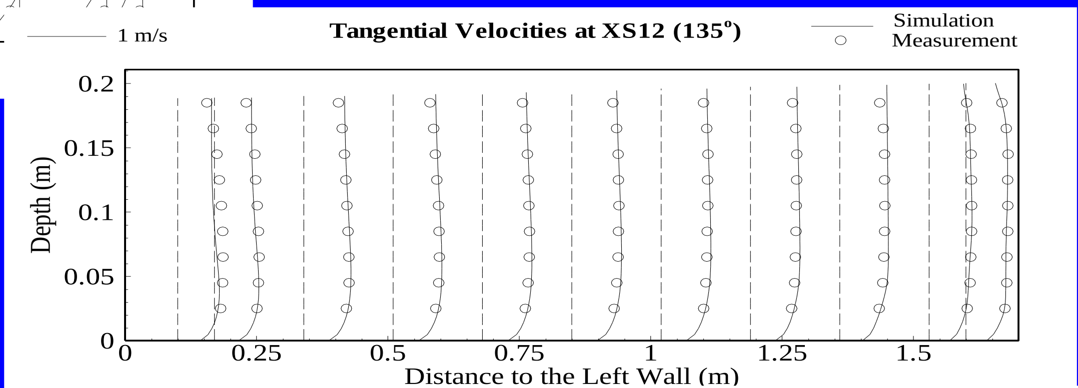


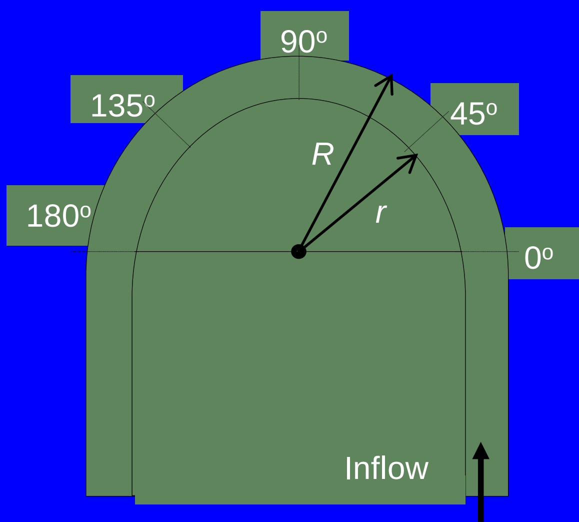

Tangential Velocities at XS09 (90°)
1 m/s
Simulation
Measurement
Depth (m)
Distance to the Left Wall (m)

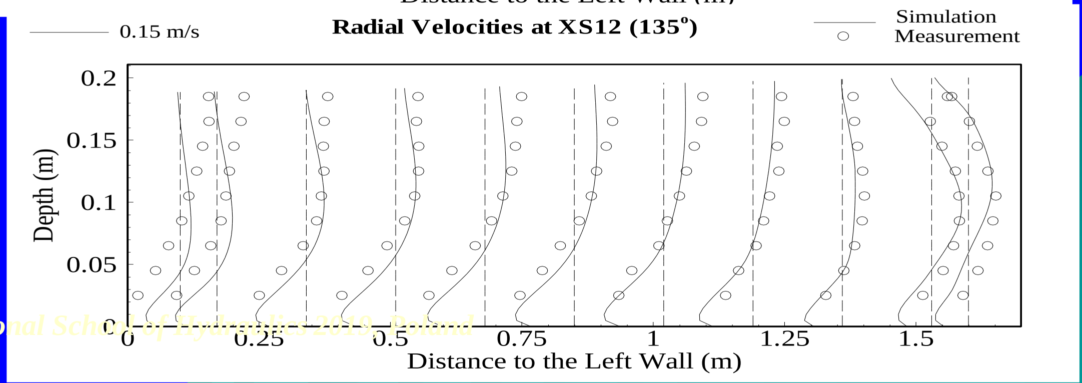



Simulation
Measurement
SUEM, SUES
DISTANCE (m)

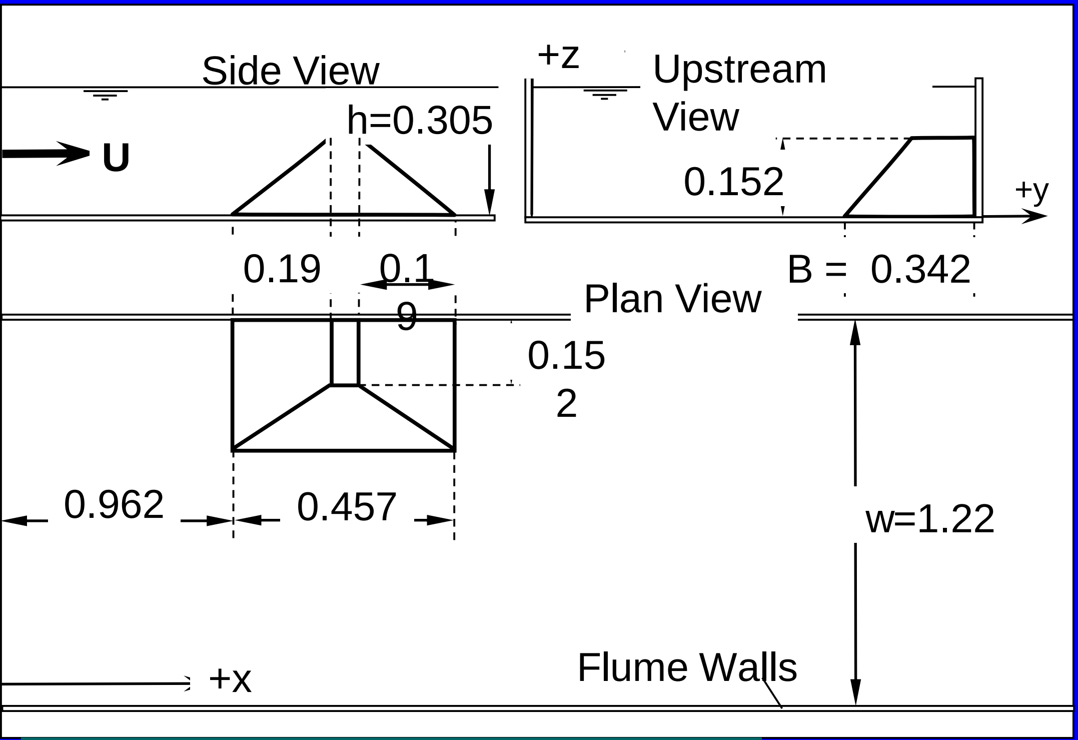

Radial Velocities at XS09 (90°)
0.15 m/s
Simulation
Measurement
Depth (m)
Distance to the Left Wall (m)


Tangential Velocities at XS12 (135°)
1 m/s
Simulation
Measurement
Depth (m)
Distance to the Left Wall (m)


90°
135°
45°
180°
0°
R
r
Inflow


Radial Velocities at XS12 (135°)
0.15 m/s
Simulation
Measurement
Depth (m)
Distance to the Left Wall (m)

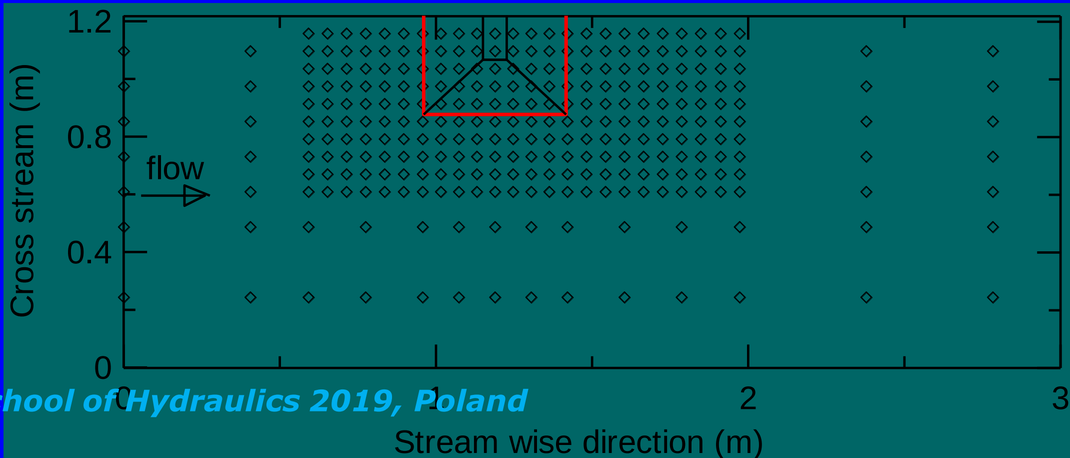
Physical Experiment of USDA

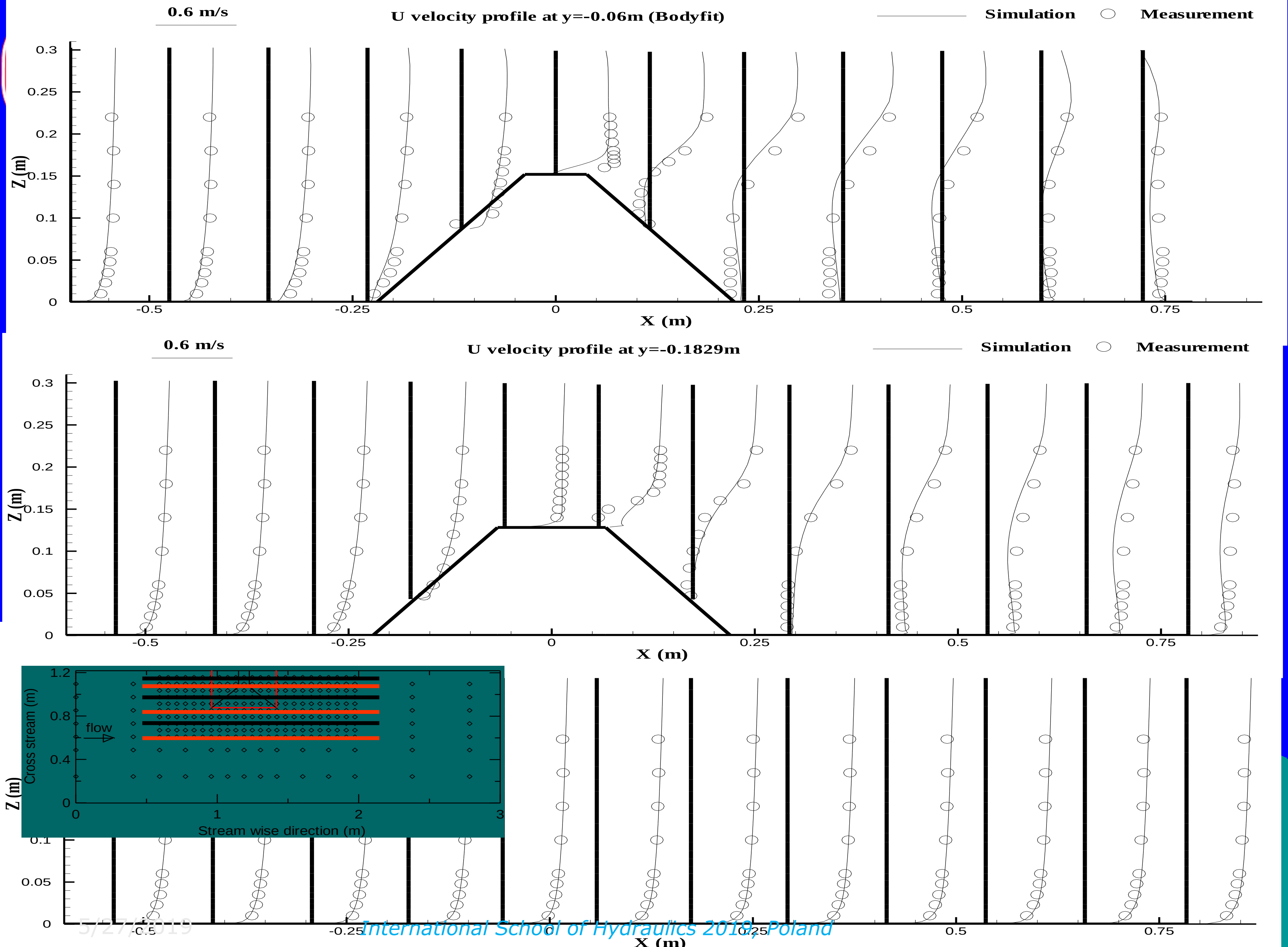


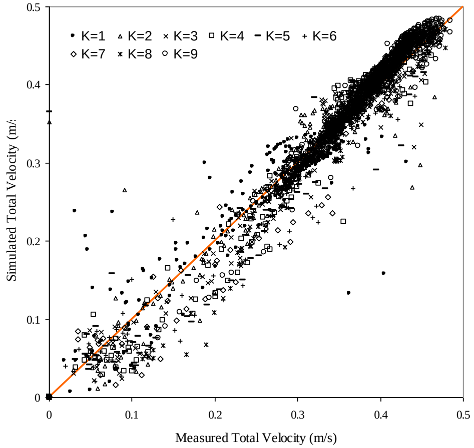
9 points in each vertical line
0.01 -0.22 m above bed
288 positions, 2592 total measurements.

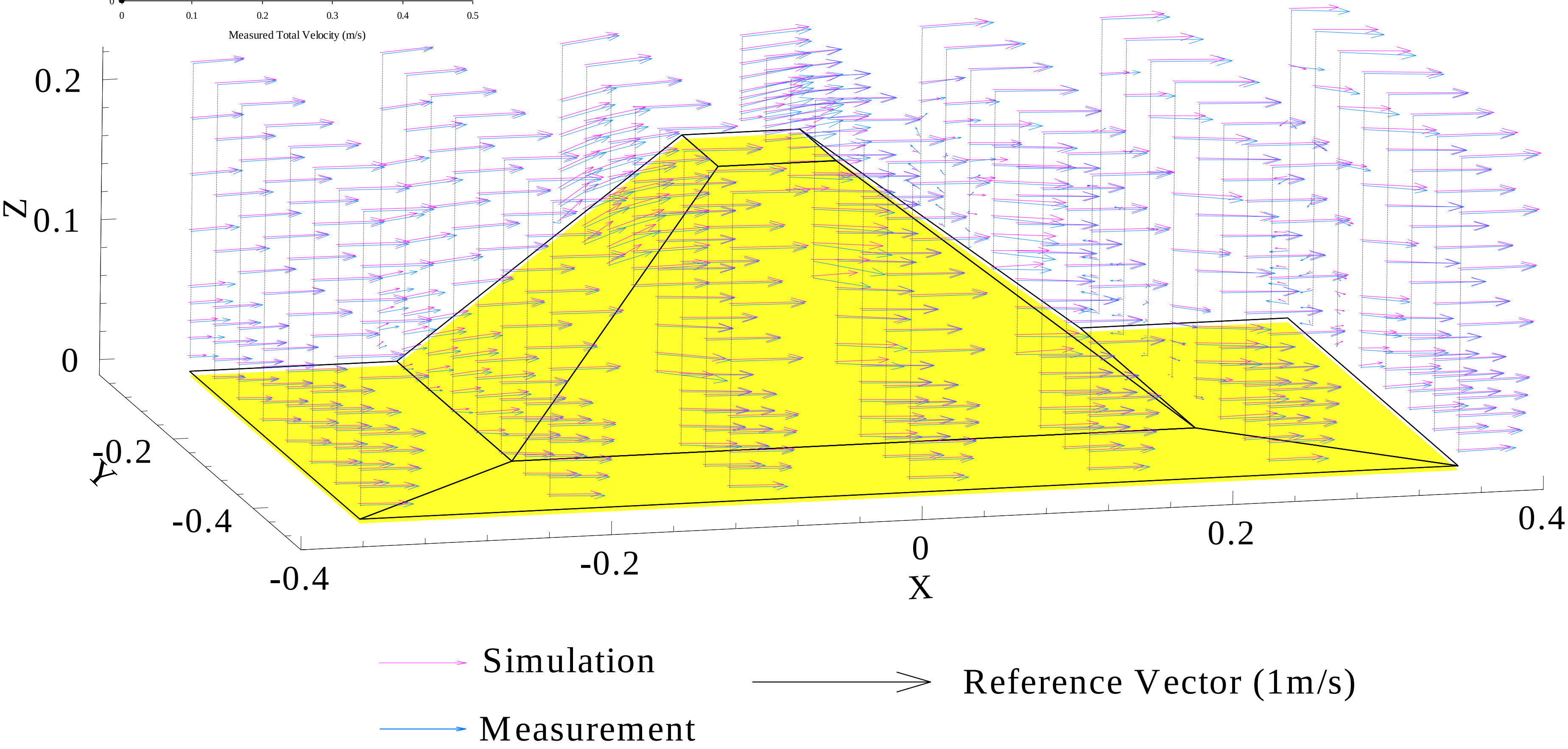
3D POINT VELOCITIES – ADV
50 Hz, 5-minute records
meas vol. (cylinder diam=6mm, 6 mm high.) = 170 mm^3

Flat fixed bed, 0.8 mm sand, cemented
Flow rate - 0.129 m^3/s
Flow depth- 0.3048 m
Mean flow velocity - 0.347 m/s
Froude number - 0.20
Shear velocity ratio (u_*/u_{*c}) - 0.7




0.6 m/s
U velocity profile at y=-0.06m (Bodyfit)
Simulation
Measurement
Z (m)
X (m)
0.6 m/s
U velocity profile at y=-0.1829m
Simulation
Measurement
Z (m)
X (m)
Cross stream (m)
flow
Stream wise direction (m)
Z (m)
X (m)

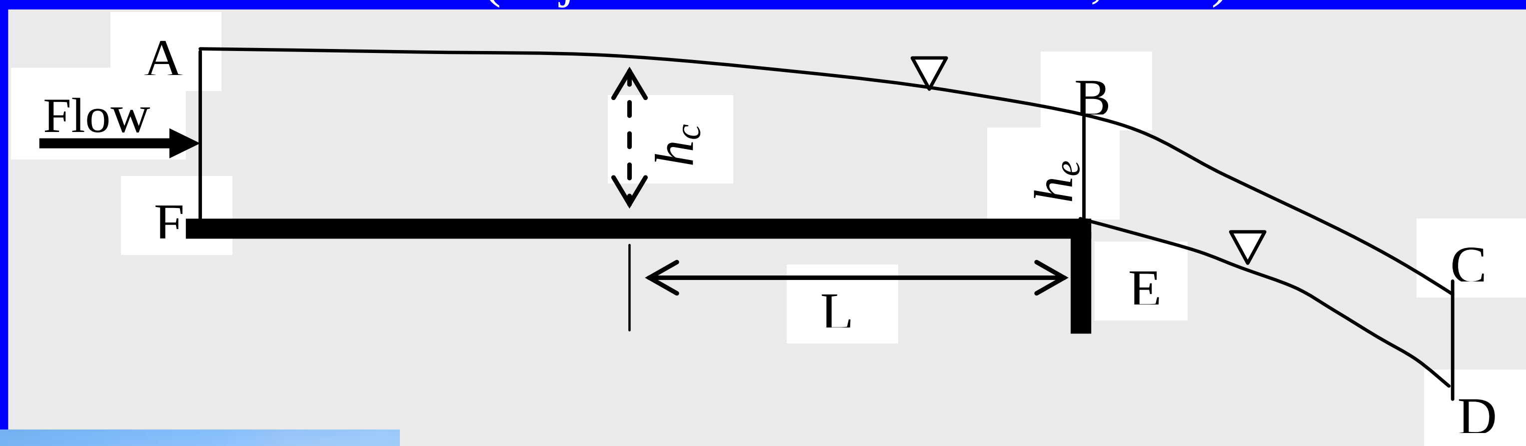

K=1
K=2
K=3
K=4
K=5
K=6
K=7
K=8
K=9
Simulated Total Velocity (m/s)
Measured Total Velocity (m/s)
0
0.1
0.2
0.3
0.4
0.5


Z
0.2
0.1
0
Y
-0.2
-0.4
X
-0.4
-0.2
0
0.2
0.4
Simulation
Measurement
Reference Vector (1m/s)

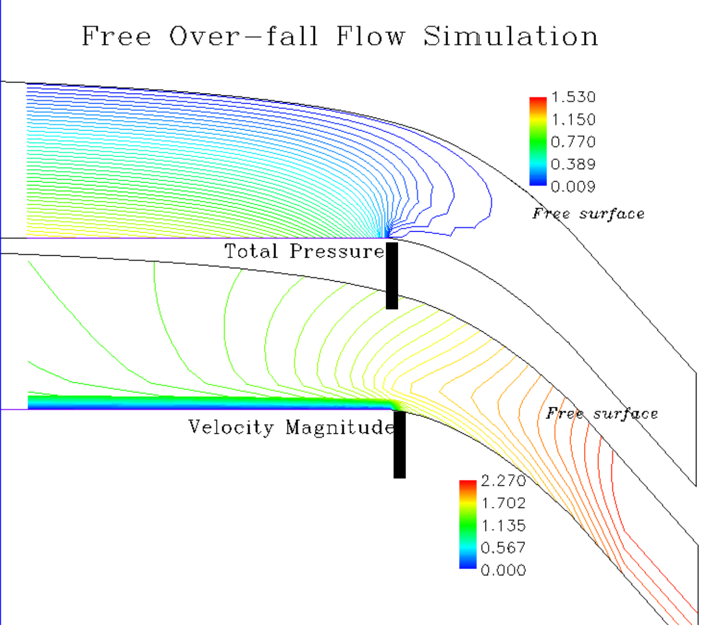
*Simulation Of A Free Overfall to validate free surface and **dynamic pressure** solution*

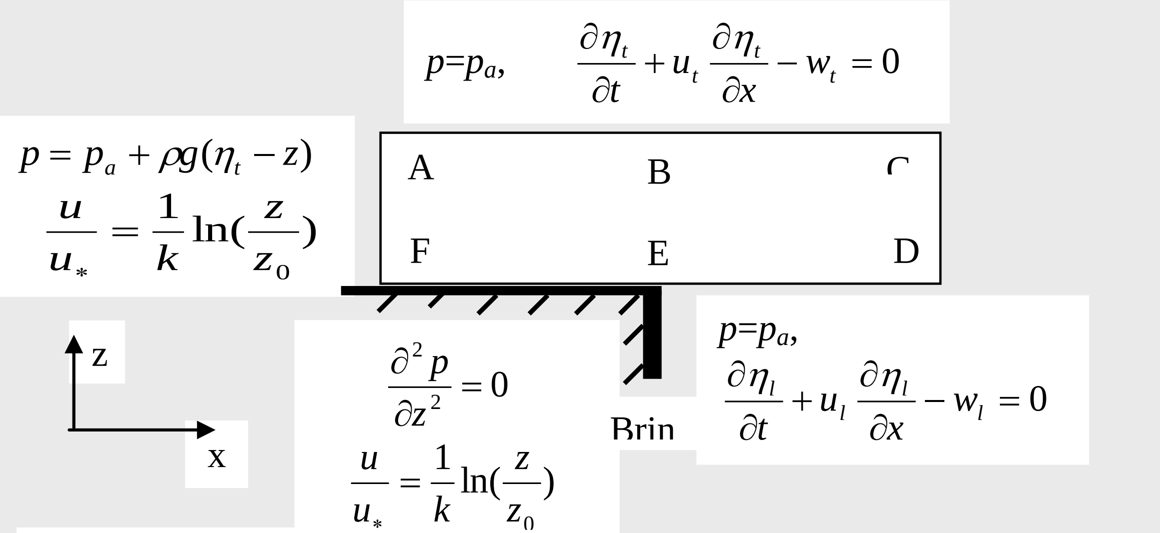
Experimental case

(Rajaratanm and Muralidhar,1968)

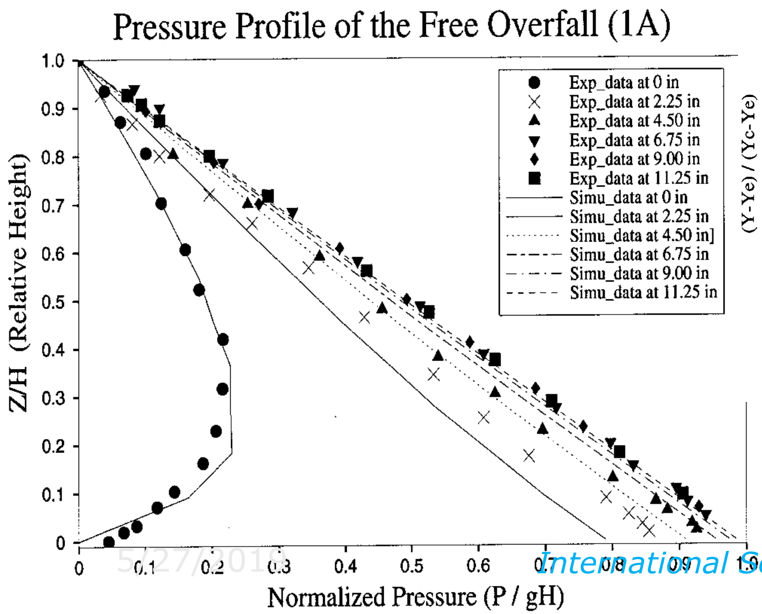


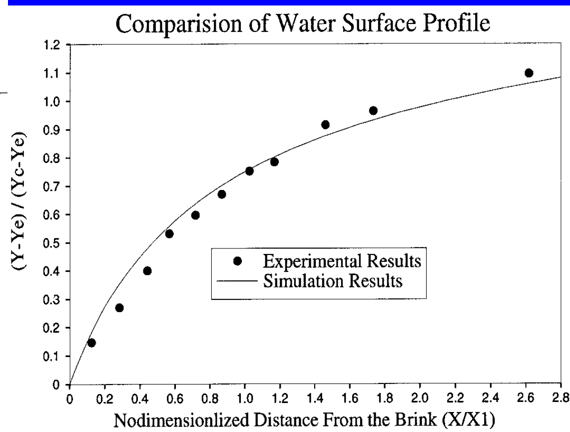
Run No.	Bed slope, *So*	Unit discharge *q* (m^2/s)	Critical depth, h_c (m)	End depth, h_e (m)	Length of overfall Z(m)
1A	0.0	0.143	0.128	0.0945	0.286

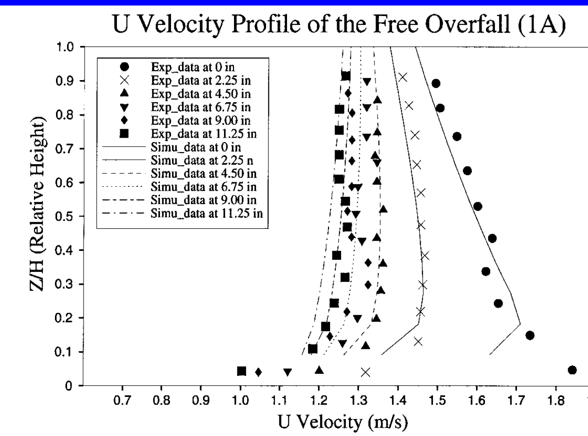




Boundary condition for free overfall flow simulation.





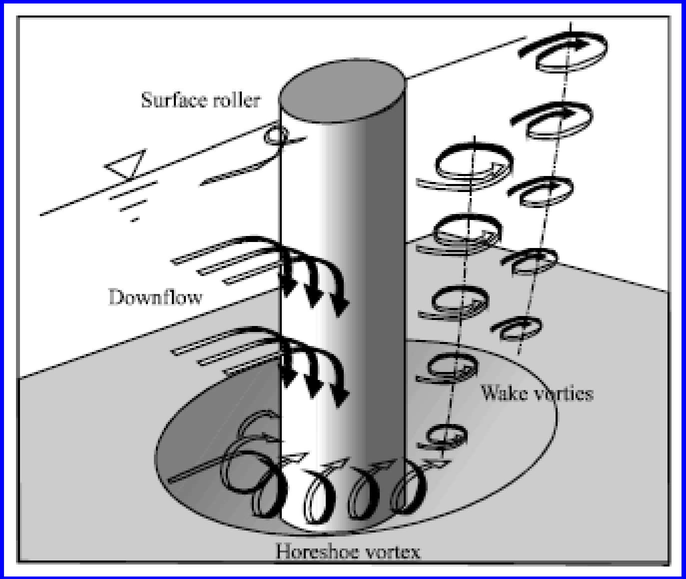


Boundary condition and simulation results of the free fall experiment

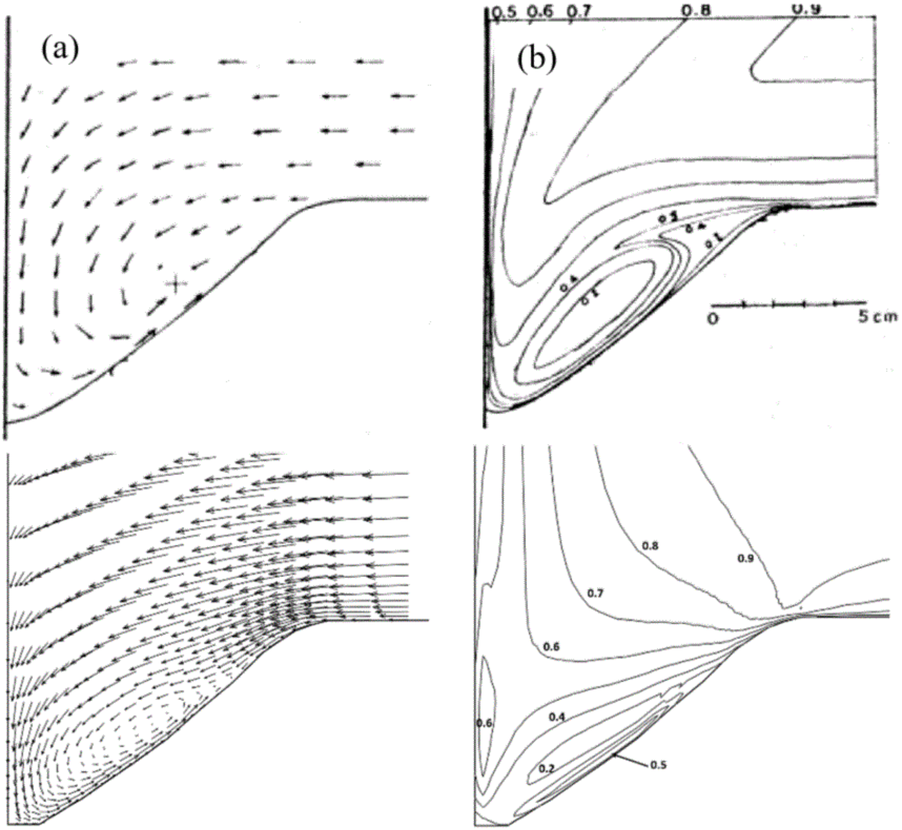
CCHE3D simulation of bridge scour.

Physical Process of Pier Scour

Sediment entrainment and transport by turbulent flow



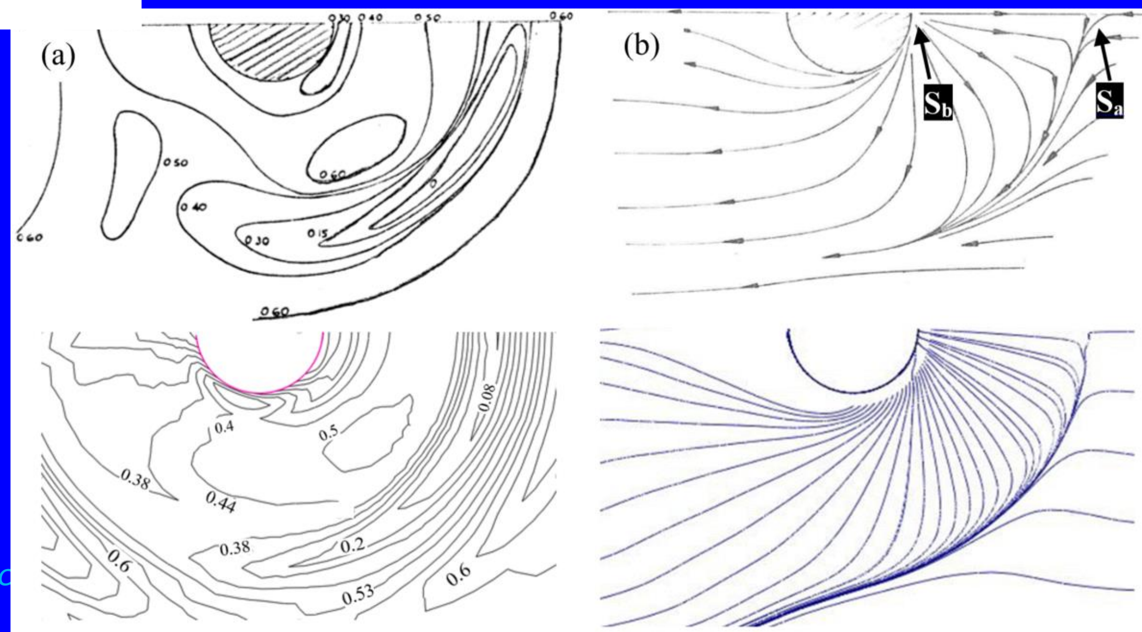
- Horse-shoe vortex flow structure
- Turbulent flow fluctuation
- Down flow near the front of the pier
- Vortices in wake zone

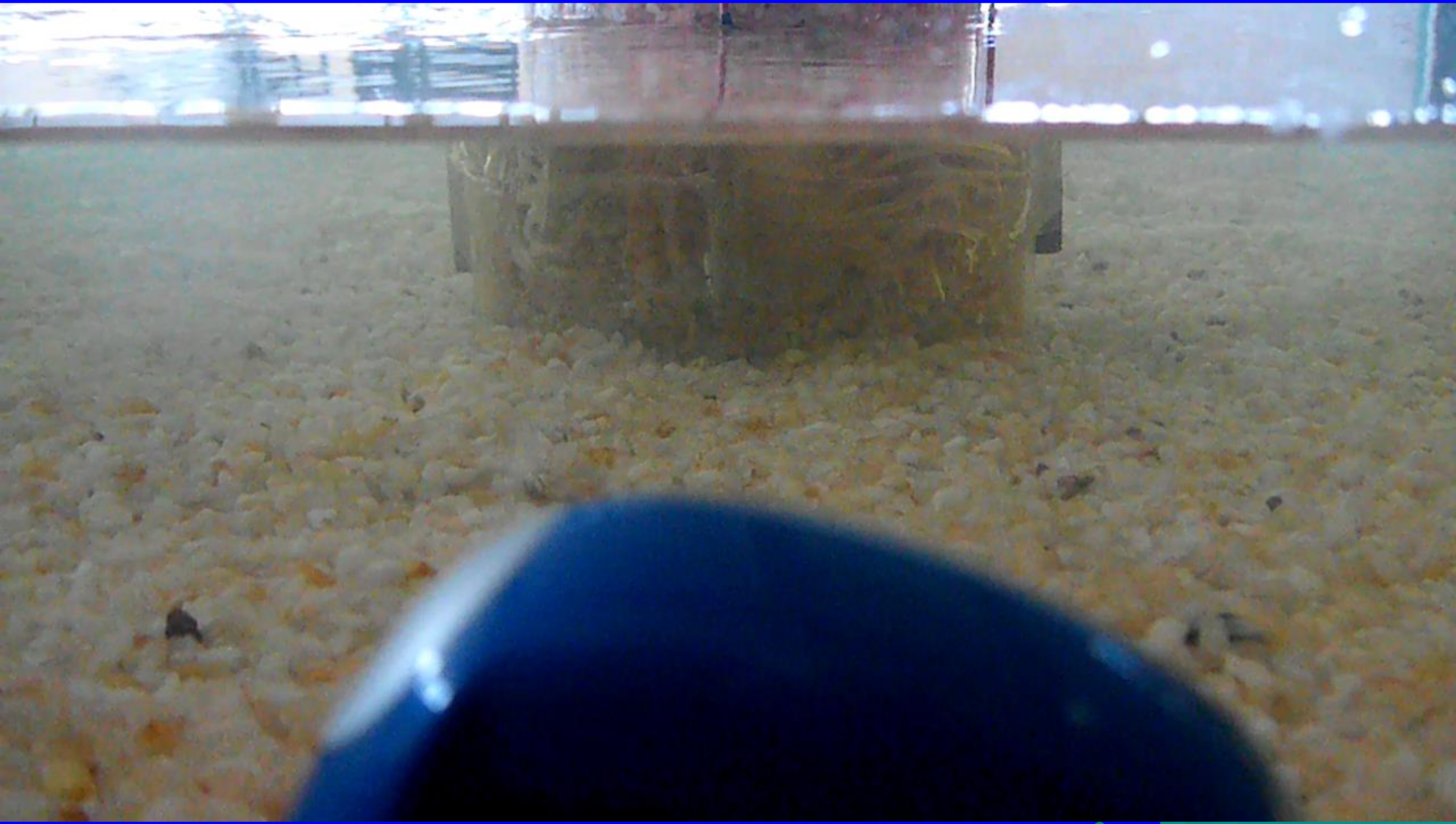


Numerical model validation using experimental data

Comparisons of simulated and measure flow velocity near the bed of the scour hole

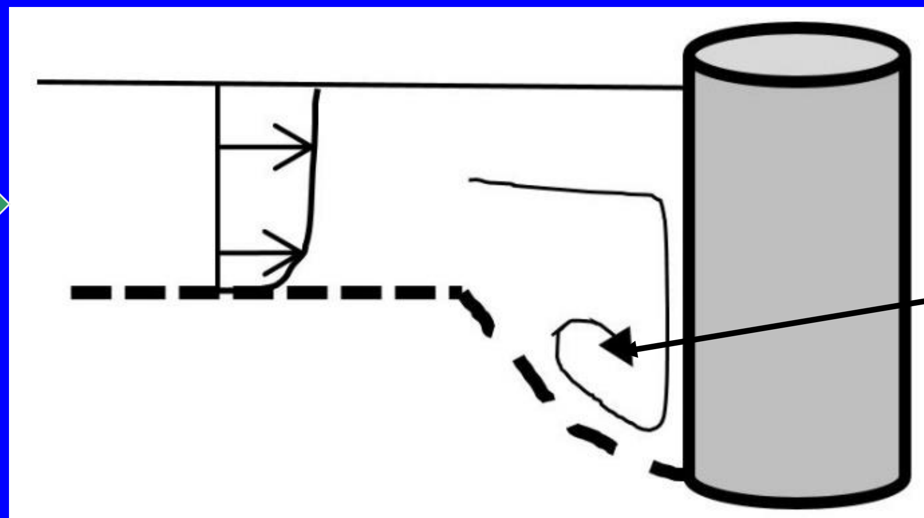
Comparisons of simulated and measure flow velocity in the vertical front plan of the scour hole





RANS models cannot produce turbulent fluctuations induced by downflows

Turbulence kinetic energy



Additional Turbulence kinetic energy generated

$$\tau = \tau_{parallel\ \ flow} + \tau_{downflow\ induced\ effective\ shear}$$

$\tau_{parallel\ \ flow}$ → Law of the wall

$\tau_{downflow\ induced\ effective\ shear}$ → Turbulent flow around structure

Difficulties in modeling local scouring

- Turbulence fluctuations
- Vertical flows
- Local vortices

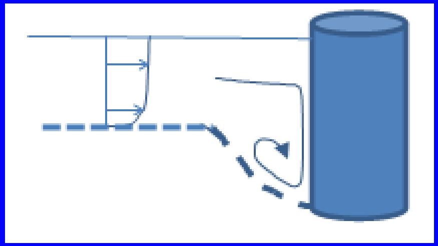
$$\tau_{effective} = \tau_{parallel\ \ flow} + \tau_{downflow\ impingement}$$

DNS/LES model
Sediment transport formulation

Local Scour Model (1)

In the approach flow

Turbulence energy (Nezu and Nakagawa,1993)



$$u' = 2.30 u_{*R} \exp(-\varsigma)$$

$$v' = 1.63 u_{*R} \exp(-\varsigma)$$

$$w' = 1.27 u_{*R} \exp(-\varsigma)$$

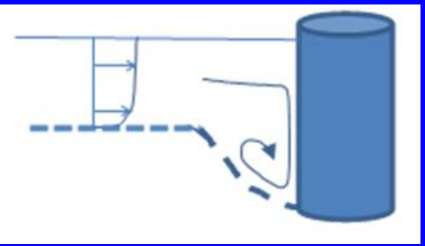
$$k = \frac{1}{2} U'^2 = \frac{1}{2}(u'^2 + v'^2 + w'^2) = 4.78 u_{*R}^2 \exp(-2\varsigma)$$

Eddy Viscosity

$$\nu_t = \kappa u_{*R} z(1 - z/h) = \kappa u_{*R} h\varsigma(1-\varsigma)$$

Energy Dissipation

$$\varepsilon_u = c_\mu \frac{k^2}{\nu_t} = c_\mu \frac{4.78^2 \exp(-4\varsigma)}{\kappa h \varsigma (1-\varsigma)} u_{*R}^3$$



Local Scour Model (2)

In the scour hole

Eddy viscosity

$$\nu_t = \ell^2 \sqrt{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i}}, \qquad \ell = \kappa h \varsigma \sqrt{1-\varsigma}$$

Turbulence

Energy

$$k = 4.78 u_{*R}^{1.5} \exp(-2\varsigma) \sqrt{\kappa h \varsigma \sqrt{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i}}}$$

Total Turbulence

Energy available

$$\overline{k}_R = 4.78 u_{*R}^{1.5} \sqrt{\kappa h} \int_0^{0.5} \left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i} \right]^{0.25} \exp(-2\varsigma)\sqrt{\varsigma}\, d\varsigma$$

Local Scour Model (3)

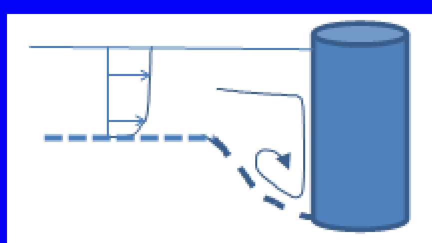
In the scour hole

Turbulence fluctuation (intruding)
for sediment entrainment
is related to available fluctuation and
near bed perpendicular velocity

$$\overline{w'^2_\perp} = \overline{w'^2_{\perp a}} \left| R_\perp \right|$$

$$R_\perp = \frac{w_\perp}{\sqrt{u_\zeta^{\,2} + v_\zeta^{\,2} + w_\perp^{\,2}}}$$

$$w_\perp = \frac{u_\zeta \dfrac{\partial \zeta}{\partial x} + v_\zeta \dfrac{\partial \zeta}{\partial y} - \overline{w}}{\sqrt{\left(\dfrac{\partial \zeta}{\partial x}\right)^2 + \left(\dfrac{\partial \zeta}{\partial x}\right)^2 + 1}}$$



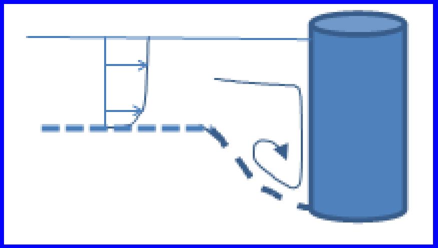
Local Scour Model (4)

The additional shear velocity for sediment incipient motion

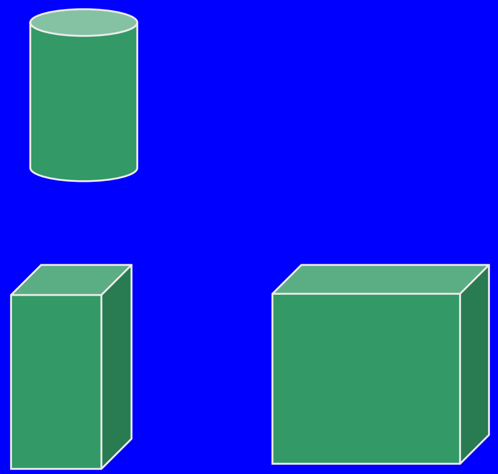
$$u_{*I} \approx \frac{C_s}{\vartheta} w'_{\perp} = \frac{C_s}{\vartheta} \sqrt{\tfrac{2}{3} \overline{k}_R \left| R_{\perp} \right|} = 0.456 C_s \sqrt{\overline{k}_R \left| R_{\perp} \right|}$$

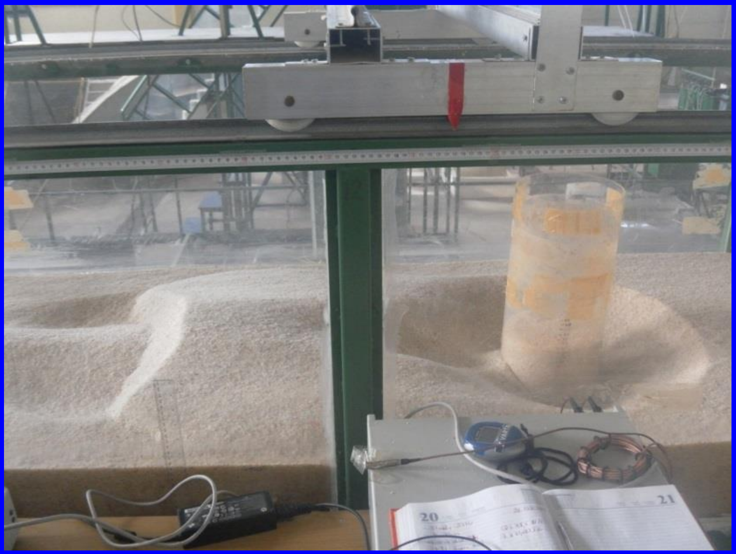
The effective shear stress:

$$u_{*e} = u_{*\square} + u_{*I}$$



Experiment Data





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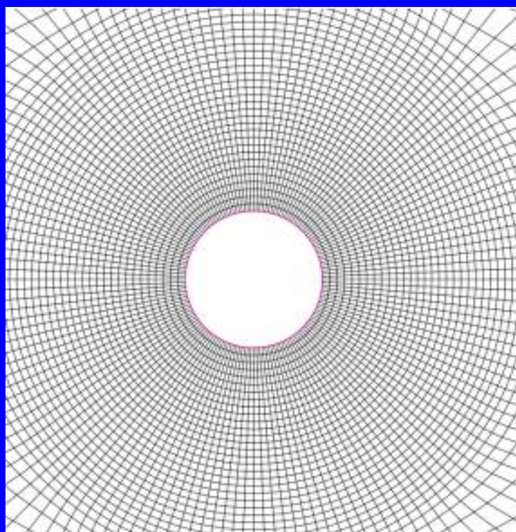
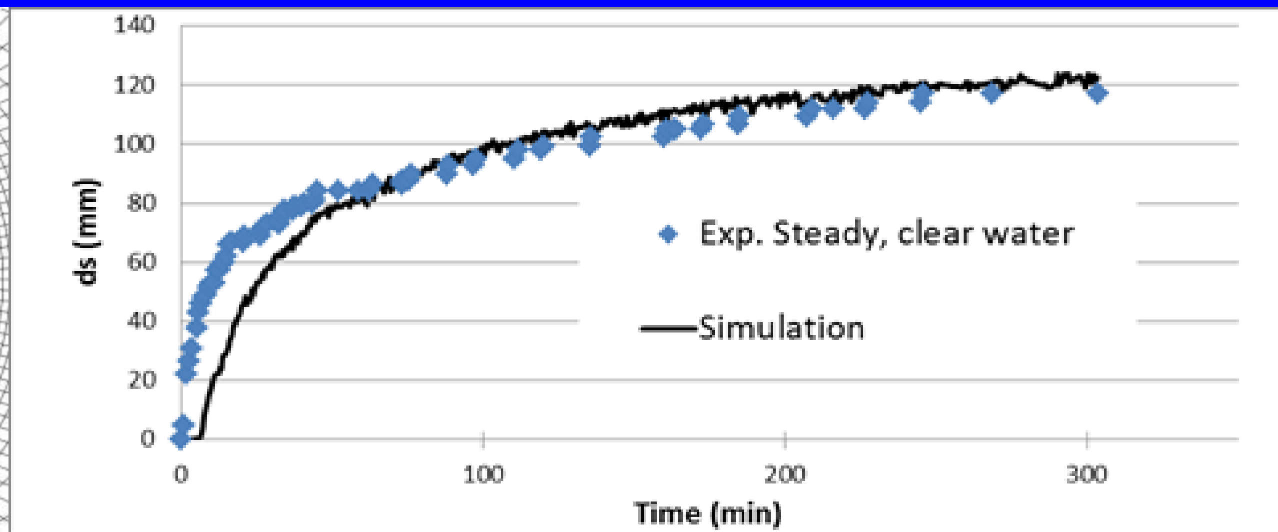
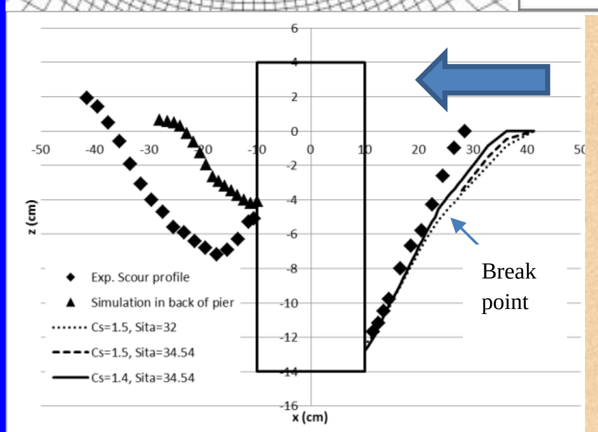
Izmir, Turkey

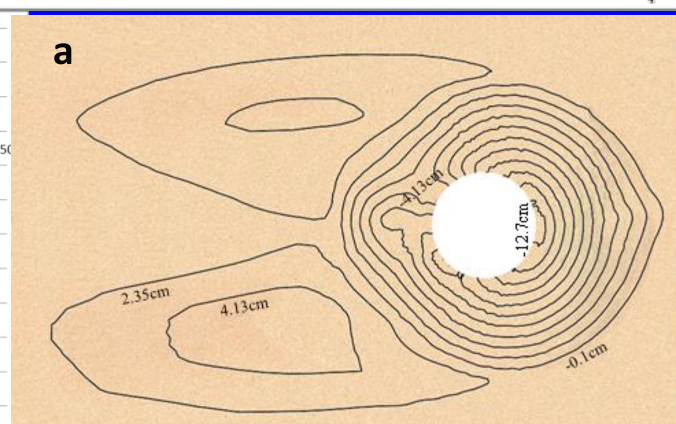
	Bed Slope	Width(m)	D(m)	d_{50}(mm)	H(m)	Q(m^3/s)	Scour time(min)	Scour depth(m)	Bed roughness k_s(mm)	Mesh
C_1	0.006	0.8	0.2	1.633	0.235	0.0585	300	0.124	1.633	41x153x12
C_2	0.006	0.8	0.074	3.4			13.3	0.06	3.4	41x153x14
C_3	0.0	1.5	0.1x0.1	1.63	0.1	0.06	4800	0.222	1.63	91x113x10
C_4	0.0	0.8	0.08x0.16	1.5	0.1	0.04	180	0.115	1.5	59x105x10

Case 1 (steady flow, uniform sediment) results



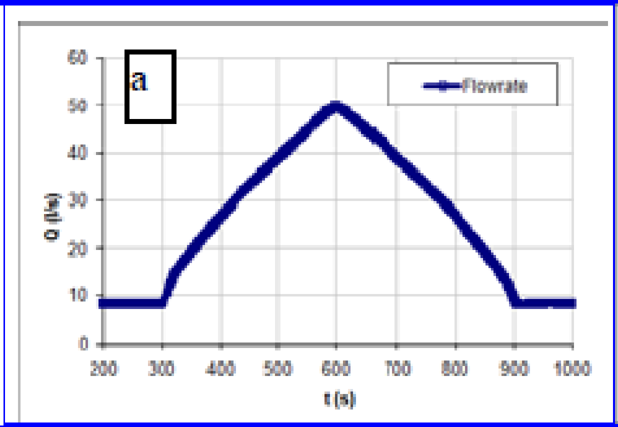




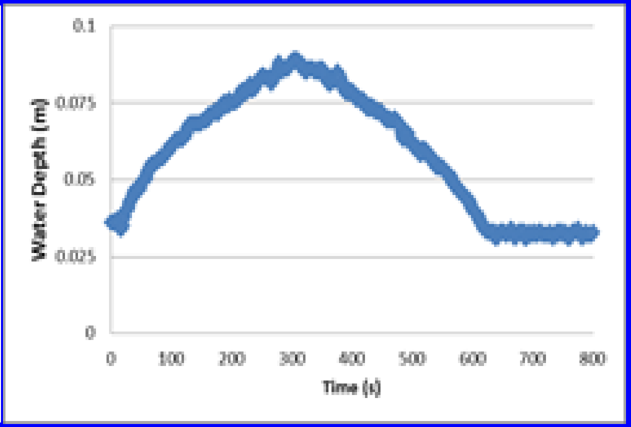


Case 2 (unsteady flow, non-uniform sediment) results

Two mode sediment

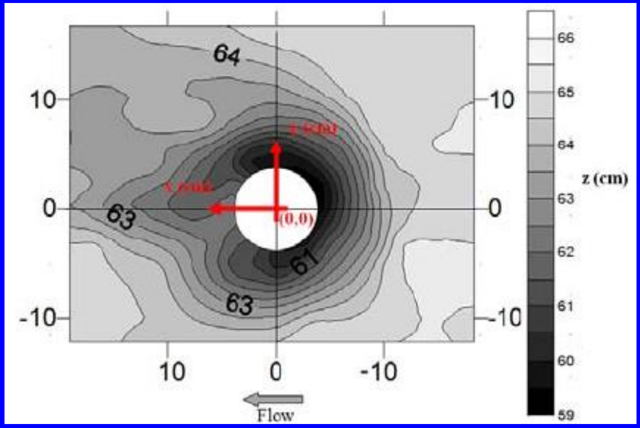


Discharge hydrograph

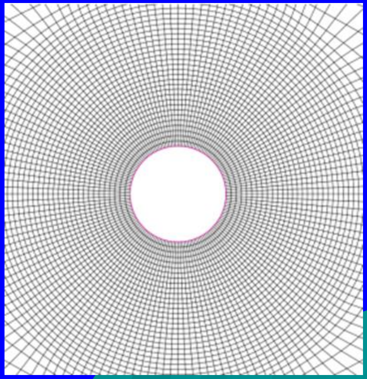


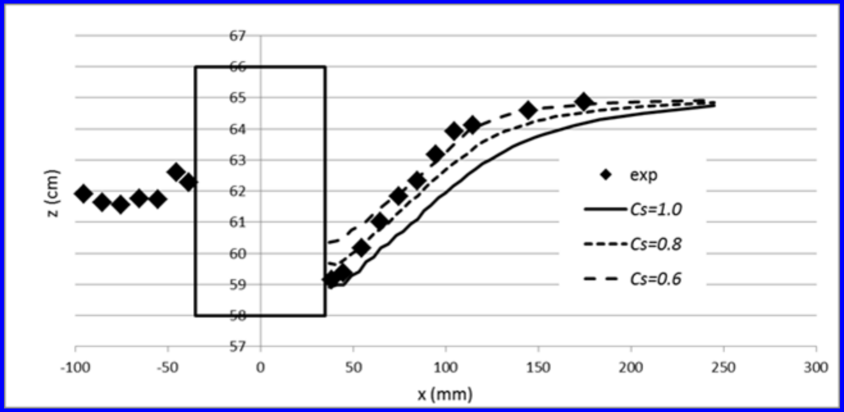
Water depth hydrograph

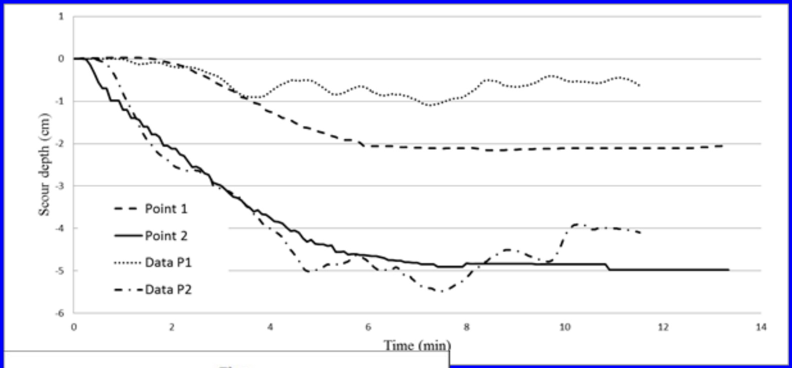
Observed
scour contours

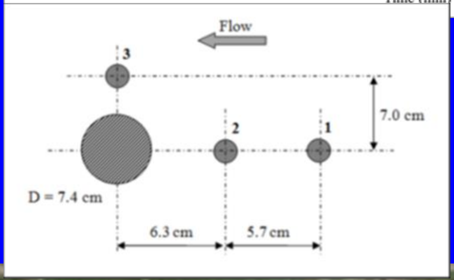


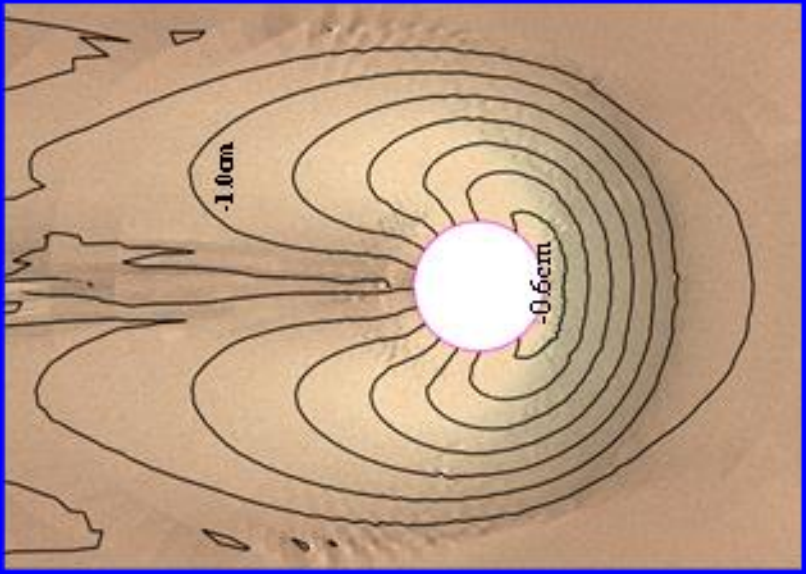
Body
fitted
circular
meth




z (cm)
x (mm)
exp
Cs=1.0
Cs=0.8
Cs=0.6
67
66
65
64
63
62
61
60
59
58
57
-100
-50
0
50
100
150
200
250
300


Scour depth (cm)
Time (min)
Point 1
Point 2
Data P1
Data P2

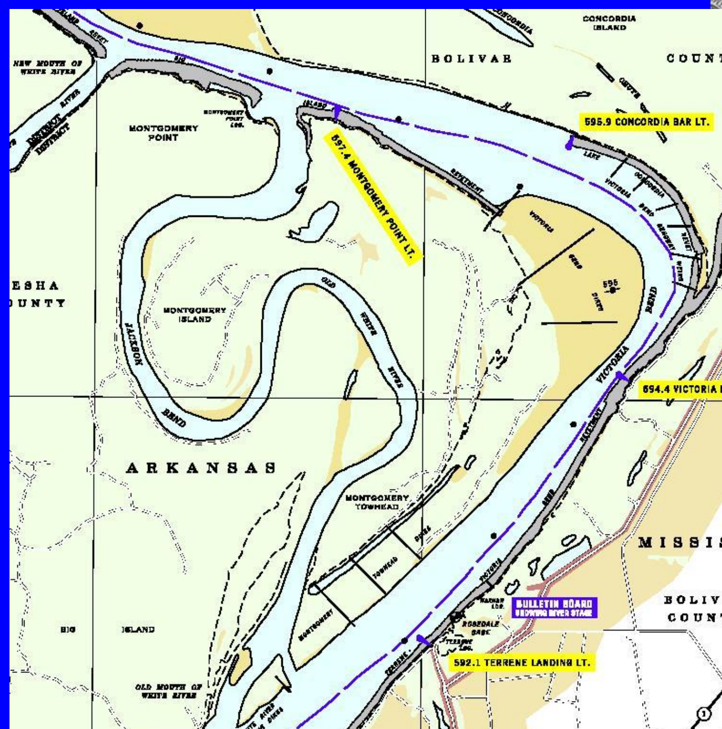

Flow
3
2
1
7.0 cm
D = 7.4 cm
6.3 cm
5.7 cm


-1.0cm
-0.6cm

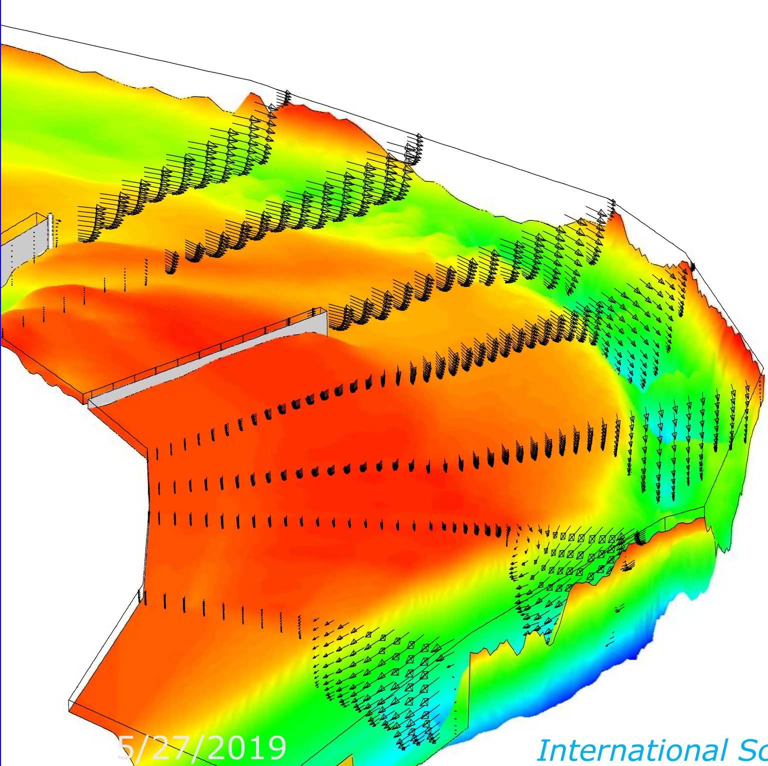
III. Application Site Validation

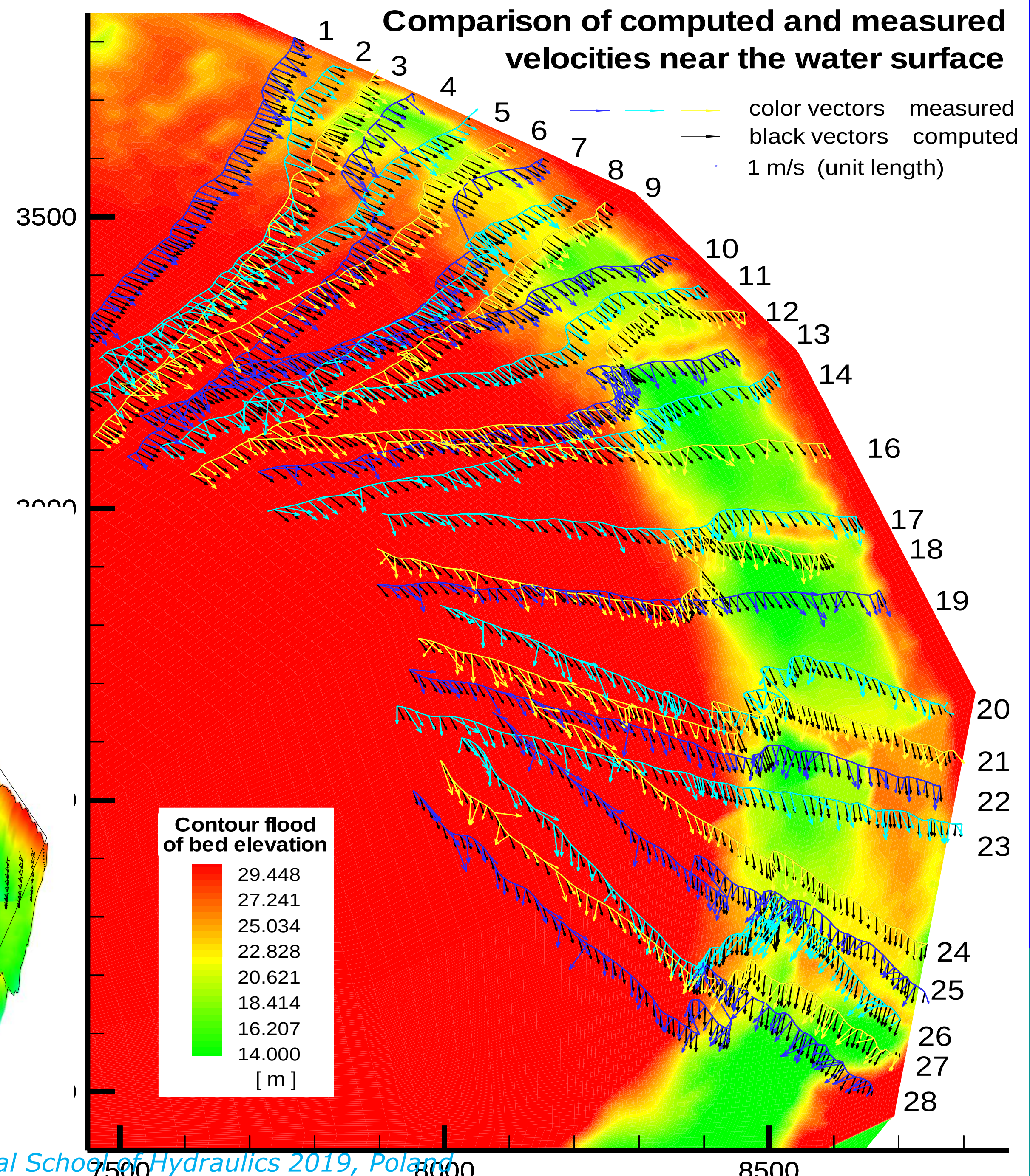
Field data of the Mississippi River was used
--Victoria Bendway

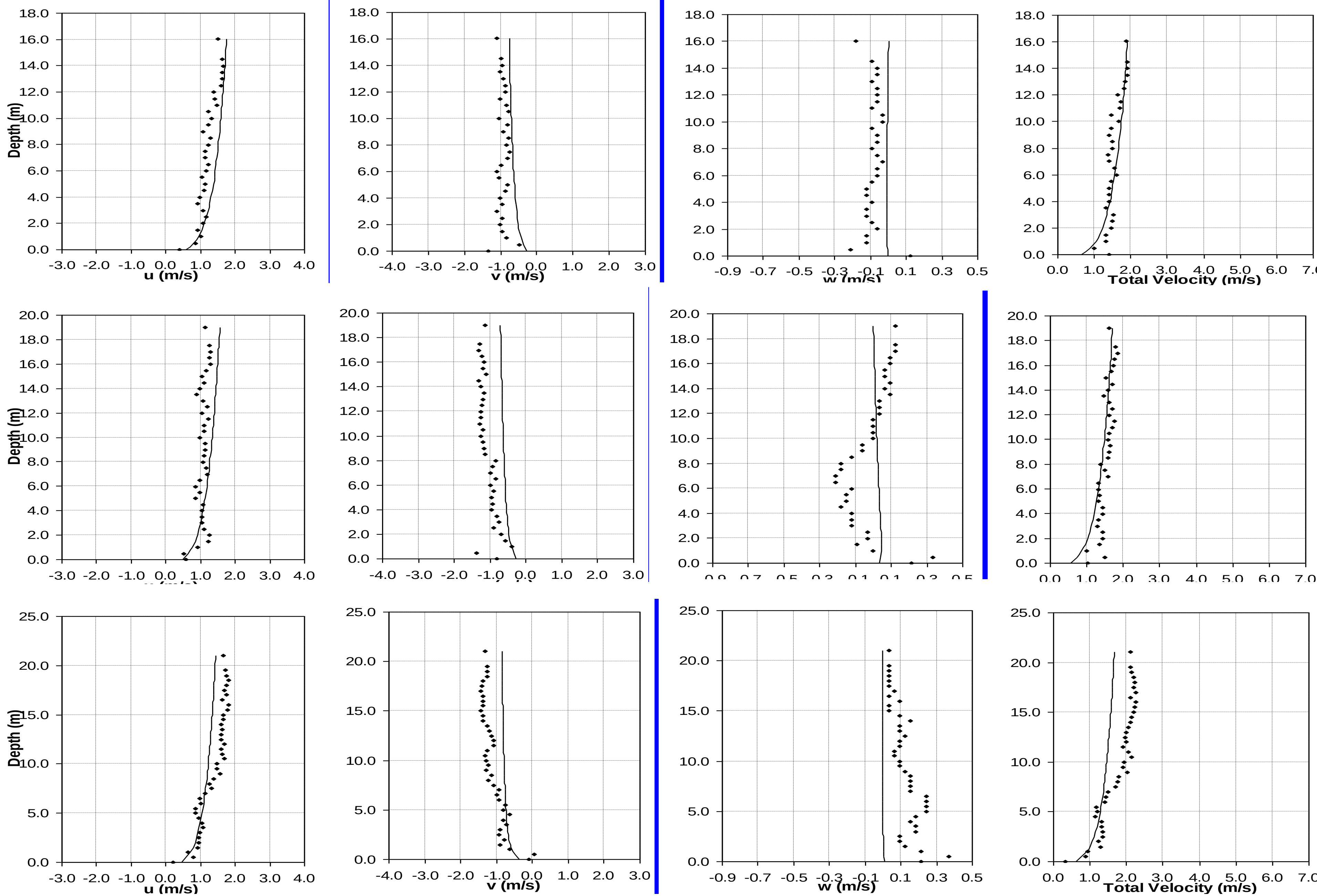
A bendway with man-made structures to improve navigation



The channel flow was simulated using real conditions. numerical model was validated with many data points and later used for hydraulic analysis







Comparison of simulation results and field measurements