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Recent trends in Environmental Hydraulics

A free surface-immersed boundary-Lattice Boltzmann method for porous-media flows

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Outlines

- Motivations and the model
- Free surface-LBM at a glance
- FS-Immersed boundary-LBM
- A seepage solution with FS-IB-LBM
- Numerical experiments
- Examples (homo-/heterogeneous)
- Conclusions
- Acknowledgements

Notes: LBM – Lattice Boltzmann method
FS – Free surface , IB – immersed boundary

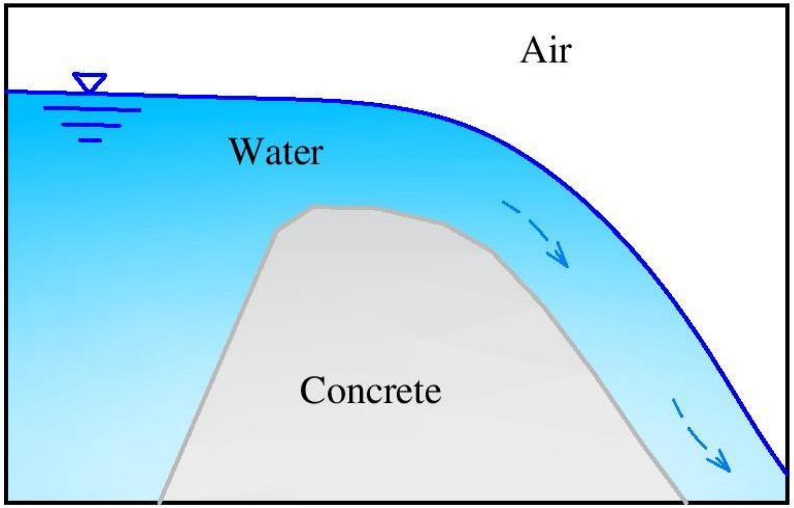
Motivations and the model

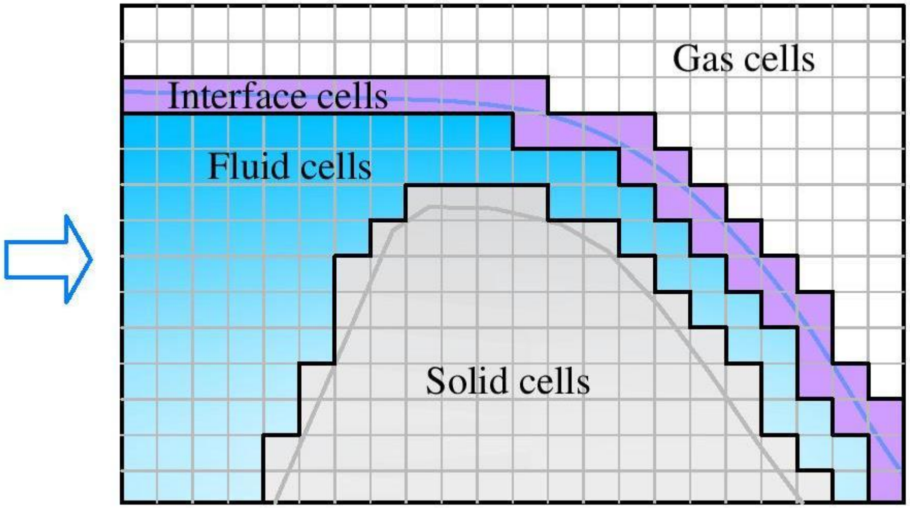
- Simple solution to porous-media flows (alternative to the NSE with Brinkman-Forchheimer extended Darcy model)
- Extending applicability of the LBM to hydraulics
- Immersed boundary method for moving porous object (vegetation, porous object in flow)

2D Free surface-lattice Boltzmann	**← Immersed boundary modification**

Notes: Porous-media flow ←→ Seepage flow
LBM – Lattice Boltzmann method
NSE – Navier-Stokes equation.

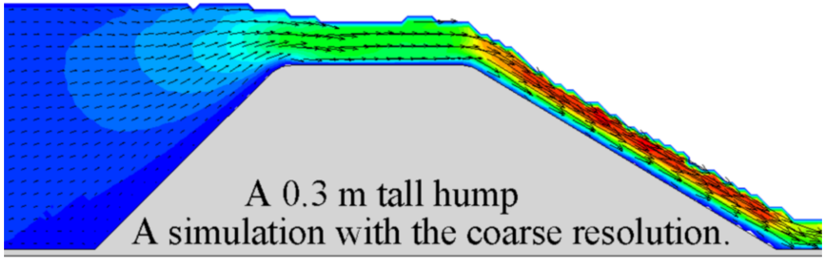
- Single phase, 2D, mesoscopic LBM-BGK model





Solves: $\frac{\partial m}{\partial t} + \mathbf{u} \cdot \nabla m = 0 \rightarrow$

$$m(t+\Delta t) = m(t) + \sum_{i=1}^{8} \Delta m_i(t+\Delta t)$$



Notes: LBM – Lattice Boltzmann method

BGK – Bhatnagar–Gross–Krook operator for the LBM.

The LBM is a method to solve Boltzmann equation on the lattice.

FS-Immersed boundary-LBM

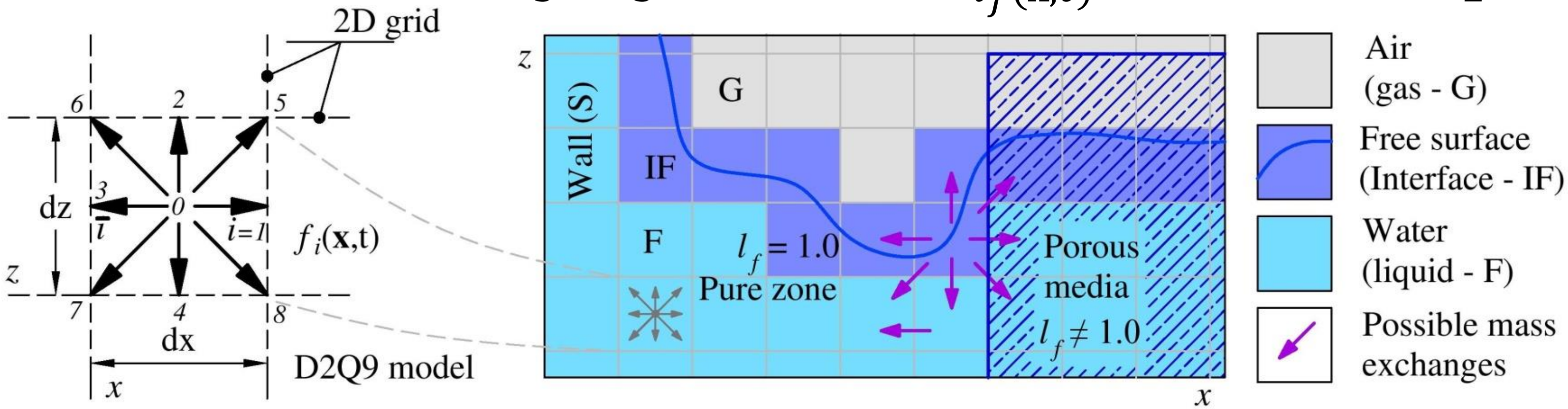
- is based on probabilistic bounce-back rule (Nobel and Torczynski, 1998).
- provides two way coupling interaction between fluid and solid
- solves:

Seepage??

$$f_i(\mathbf{x} + \mathbf{c}_i \delta t, t + \delta t) - f_i(\mathbf{x}, t) = -\frac{1-\beta}{\tau}\left(f_i(\mathbf{x}, t) - f_i^{\,eq}(\mathbf{x}, t)\right) + (1-\beta)A_i + \beta f_i^{\,m}(\mathbf{x}, t)$$

LB equation for flow **IB modification**

- Physical variables : $p = \frac{\rho}{3} = \frac{1}{3}\sum_{i=0}^{8} f_i$ and $\frac{\rho \mathbf{u}}{l_f(\mathbf{x},t)} = \sum_{i=0}^{8} \mathbf{c}_i f_i + \frac{\mathbf{F}\delta t}{2}$



Notes: LB equation – discretized Lattice Boltzmann equation
FS – free surface, IB – Immersed boundary.
two way – Fluid-solid bi-directional interactions

A seepage solution with FS-IB-LBM

$$f_i(\mathbf{x}+\mathbf{c}_i\delta t, t+\delta t) = f_i(\mathbf{x},t) - \frac{\delta t(1-\beta)}{\tau}\left(f_i(\mathbf{x},t) - f_i^{\ eq}(\mathbf{x},t)\right) + \beta f_i^{\ m}(\mathbf{x},t) + \delta t(1-\beta)A_i$$

where $\beta = 1 - l_f(\mathbf{x},t)$

- Solid $l_f = 0$ → $f_i(\mathbf{x}+\mathbf{c}_i\delta t, t+\delta t) = f_i(\mathbf{x},t) - f_i^{\ m}(\mathbf{x},t)$ - Full bounce back or impervious
- Liquid $l_f = 1$ → $f_i(\mathbf{x}+\mathbf{c}_i\delta t, t+\delta t) = f_i(\mathbf{x},t) - \delta t\frac{f_i(\mathbf{x},t) - f_i^{\ eq}(\mathbf{x},t)}{\tau} + \delta t A_i$ - free flow
- Porous zone $0 < l_f < 1$ → An expression, same as one proposed by Walsh et al. 2009.

$$f_i(\mathbf{x}+\mathbf{c}_i\delta t, t+\delta t) = f_i - \frac{\delta t}{\tau}\left(f_i - f_i^{\ eq}\right) + \delta t A_i + \beta\left(f_i^{eq}(u_s) - f_i^{eq} - A_i\right)$$

- Without further exploration, the IB-LBM is applicable for porous media flows.

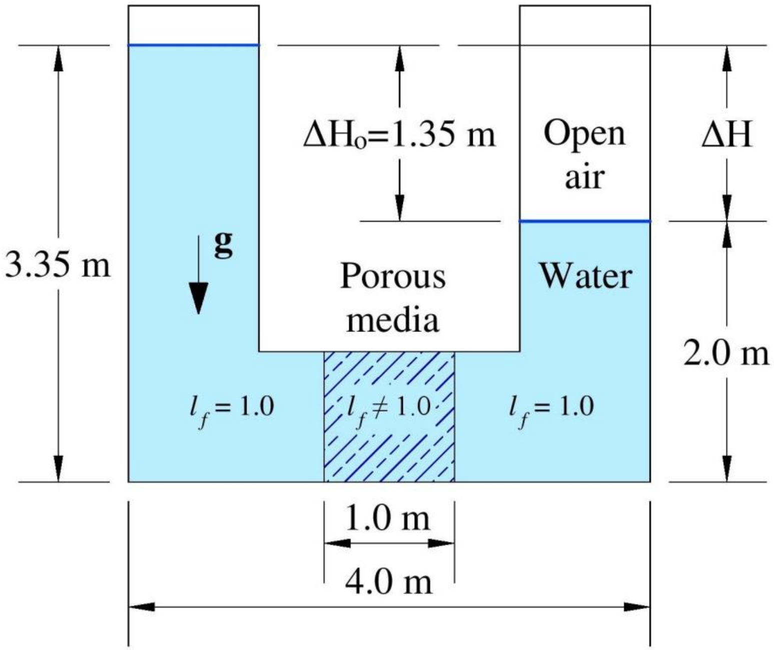
by Walsh et al. (2009)
$k = l_f \nu_L / 2(1 - l_f)$

According to Darcy's law
$K_{pR} = \mathrm{u}\nu\Delta L / g\Delta H$

$k \sim K_{pL} = K_{pR}/\Delta x^2$?

Notes: K_p - intrinsic permeability [m^2], k – model permeability [-]
R – subscript for real value, L – subscript for lattice value
K_{pL} is dimensionless intrinsic permeability in lattice.

Numerical experiments: Validation



Analytical solution to flows trough U-tube with porous media:

- $\Delta H = \Delta H_o \exp(-2K_h t/L)$
- $u = \Delta H_o K_h \exp(-2K_h t/L)/L$

Parameters for FS-IB-LBM:

- 400×340 grids ($\Delta x = \Delta z = 0.01$ m)
- lattice gravity $\mathbf{g}_L = 1.28 \times 10^{-4}$, which gives the relaxation time $\tau = 0.5315$

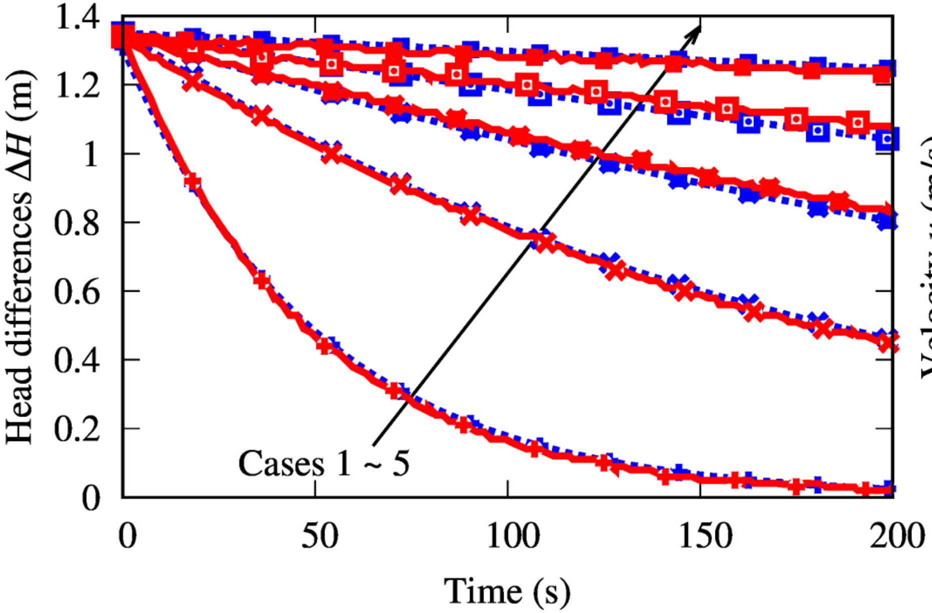
Analytical solution = Numerical results

To define the relation: $K_{pL} \sim k$

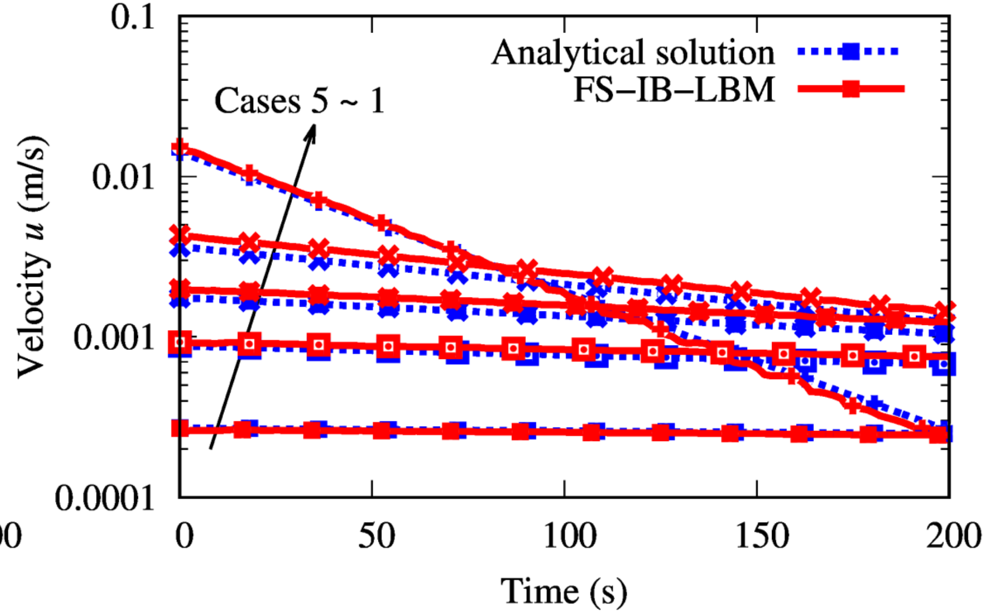
Notes: K_h - hydraulic conductivity [m/s], t – time [s]
L – length of the porous zone [m]. τ is dimensionless relaxation time of BGK model which relates to fluid viscosity.

Numerical experiments: Validation

Head differences



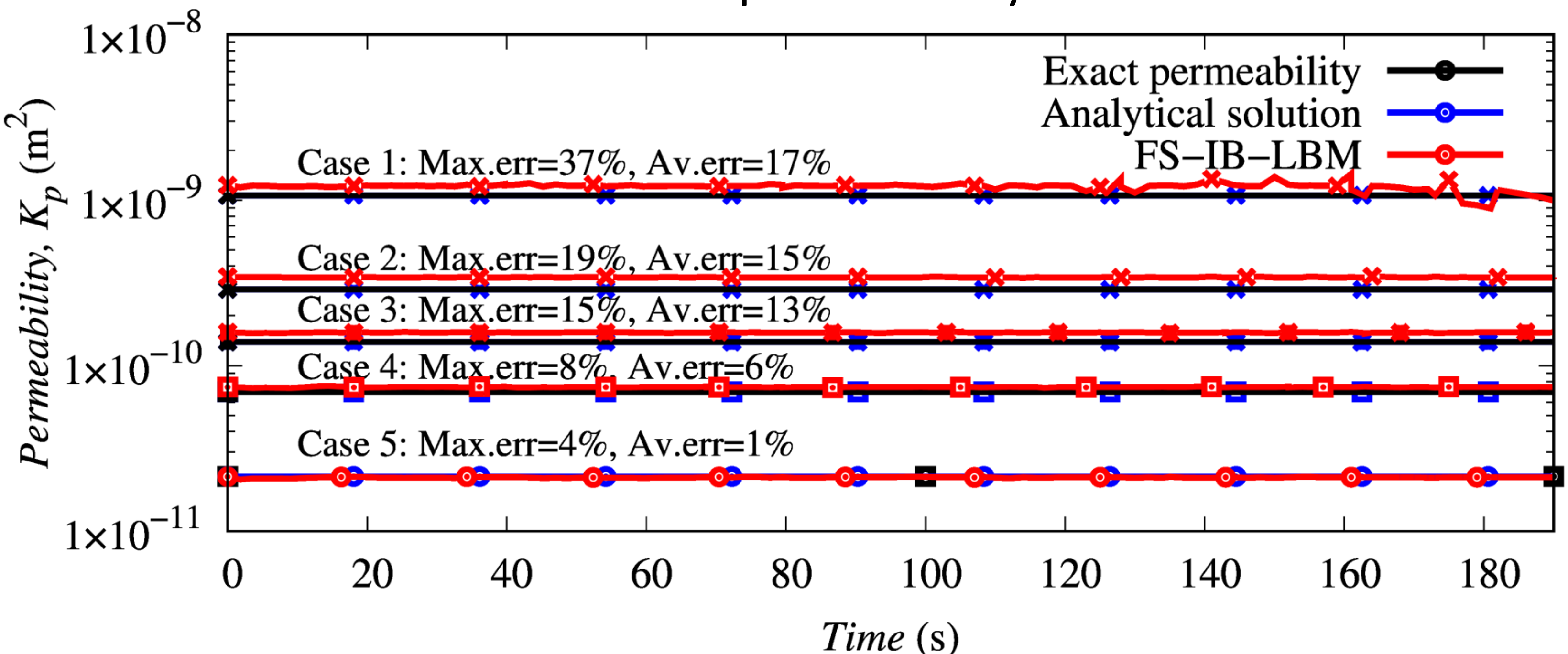
Average seepage velocity



Cases	1	2	3	4	5
Analytical solution K_{pR}, m^2	1.07E-9	2.09E-10	1.39E-10	6.96E-11	2.14E-11
l_f in FS-IB-LBM	0.9	0.7	0.5	0.3	0.1

Numerical experiments: Validation

Time series of recalculated permeability from numerical solutions



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Note: FS-IB-LBM – Free surface-immersed boundary-lattice Boltzmann method

Numerical experiments: Validation

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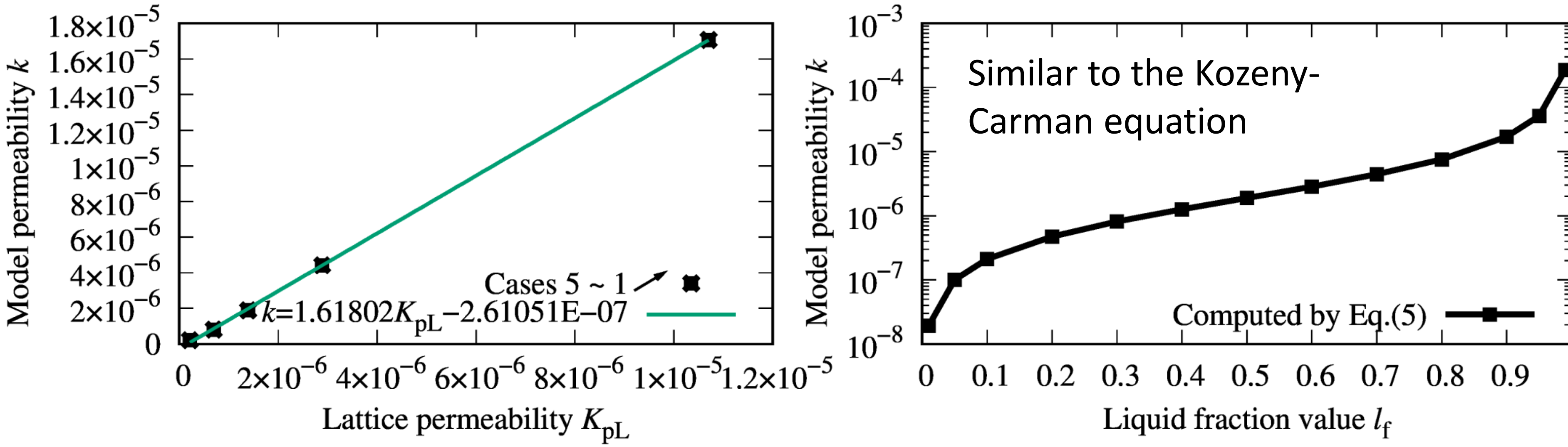

$k \cong 1.618\, K_p/\Delta x^2$ where $k = l_f \nu_L / 2(1 - l_f)$

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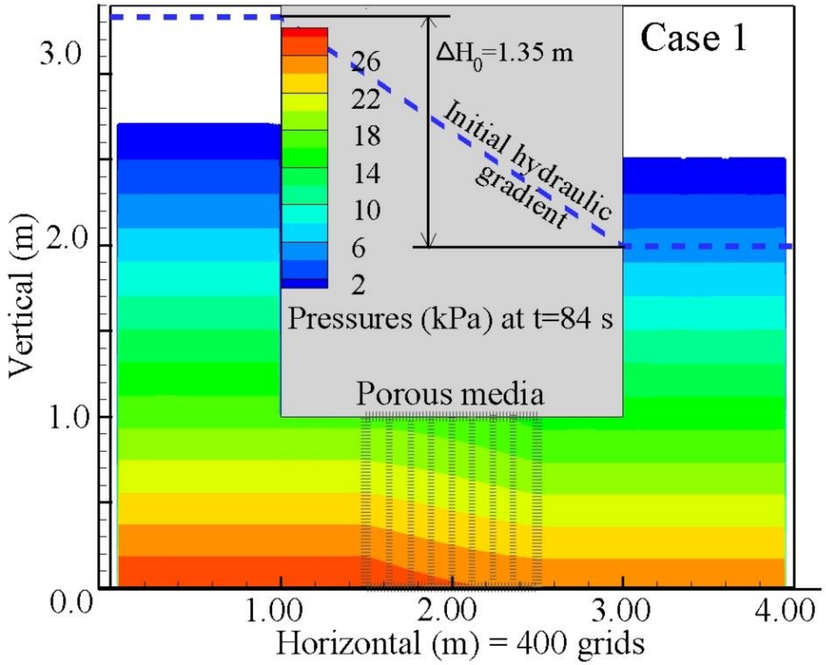
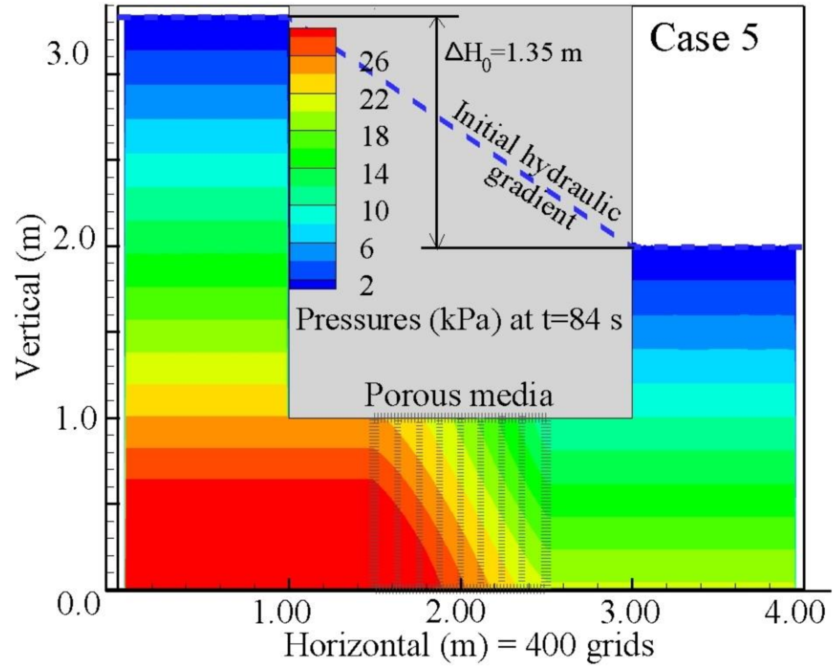
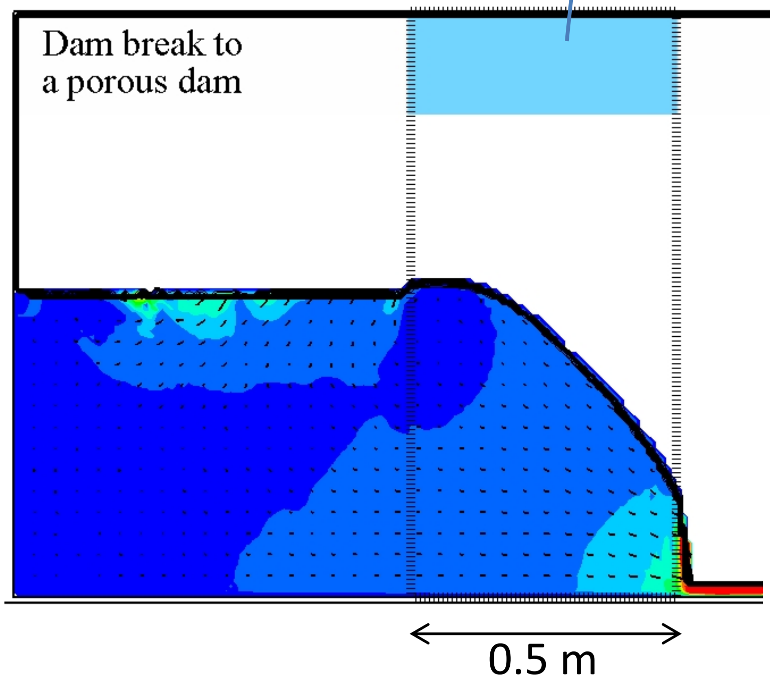




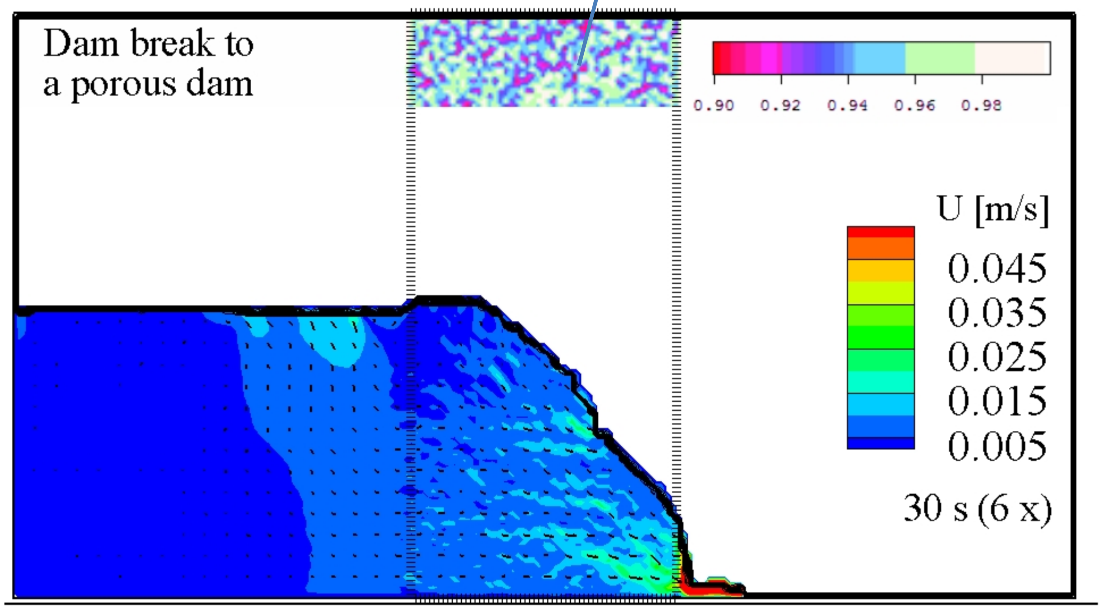
Cases	1	2	3	4	5
Analytical solution K_h, m s^{-1}	0.01	0.0027	0.0013	0.00065	0.00020
l_f in FS-IB-LBM	0.9	0.7	0.5	0.3	0.1

Examples (homo-/heterogeneous)

Homogeneous dam ($K_{pR} = 9.3 \times 10^{-10}\ m^2$)

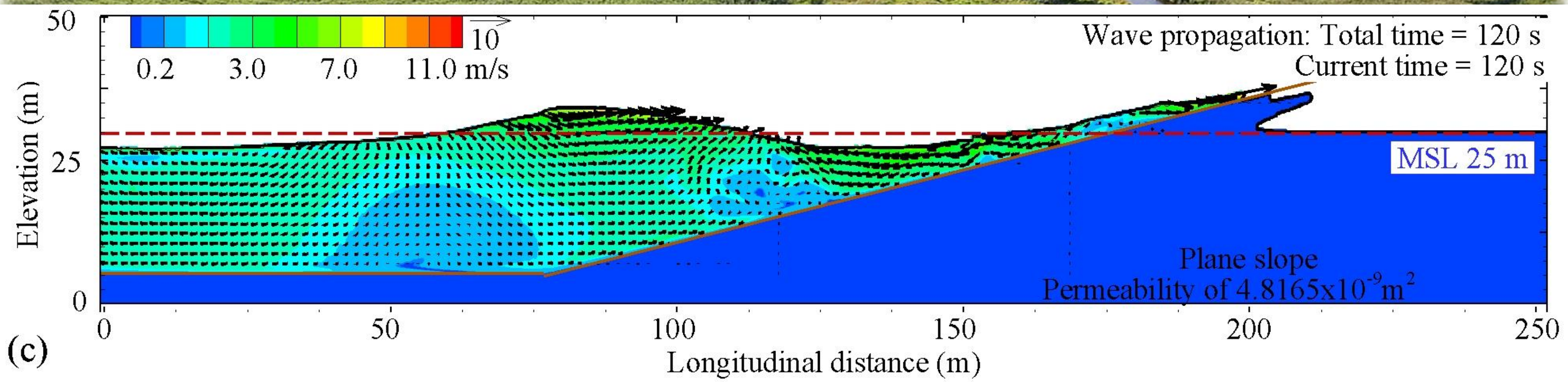


Heterogeneous dam ($K_{pR} \approx 9.3 \times 10^{-10}\ m^2$)



Examples (homo-/heterogeneous)

- Seepage through and under any hydraulic structures
- Subsurface flows and flows in vegetated channels (wetlands, lakes with emergent plants ,e.g. reeds)
- Fluid-solid (porous or pure solid, flexible solid) interactions



Conclusions

- FS-IB-LBM is introduced for flows through porous media
- The relation of permeability and scaling of the permeability are purposed ($k \cong 1.618\, K_p/\Delta x^2$).
- The model is rather simple than other alternative models and possesses advantages of immersed/moving boundary condition.
- Nevertheless, the model is at representative elementary volume scale, it is applicable for pore scale porous media.
- FS-IB-LBM retains the inherent advantages for parallelization and smooth treatment of hydrodynamic force estimation.

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Photo: Ludwig Boltzmann's grave (1844-1906) Vienna, Austria