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# Shallow Water Equations as a Mathematical Model of Whitewater Course Hydrodynamics

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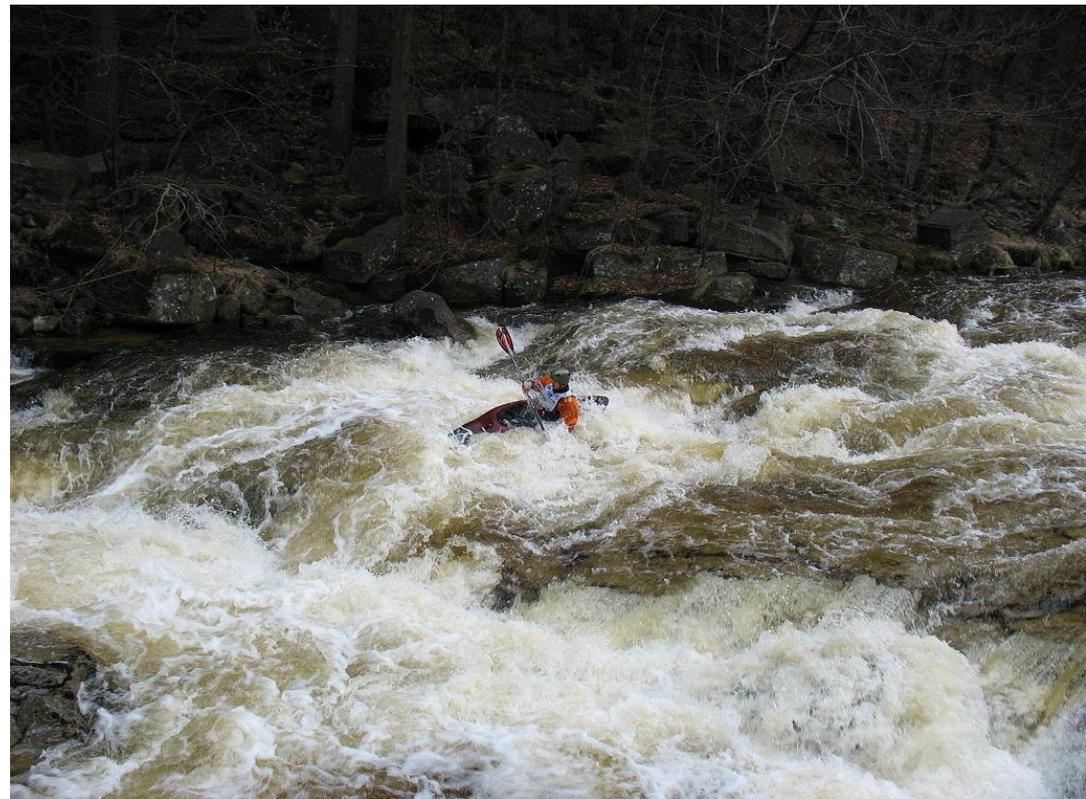
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## Water sports – whitewater kayaking and whitewater canoeing – types

- **River running is a race: canoe several miles as quickly as possible.**
- Playboating (Freestyle or Rodeo) is a more gymnastic and artistic kind of kayaking.
- Slalom is a technical competitive form of kayaking, and the only whitewater event to appear in the Olympic Games.



[Source: Wikipedia](#)



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[Source: Wikipedia](#)



## **Whitewater courses – different users, different expectations**

- **Sportsmen: they want a spectacular course, high velocities and high waves, but constant in time – the same properties and conditions for all competitors.**
- **Recreational users and beginners: they want a safe course, but varying.**
- **Sponsors: the course must be attractive to the public.**
- **Financiers: the course must be efficient and economic.**



## **Whitewater courses – general design principles**

- **minimum length of 300 m - according to use in competitions like the Olympic Games and World Championships (ICF – International Canoe Federation)**
- **simple cross-section shape - trapezoidal, U-shape**
- **profile width not less than 8 m, optimal 10-12 m - double the size of slalom boats**
- **depth - minimum 0.4 m, average depth (safe for Eskimo rolls) approx. from 0.75 m to 0.9 m, for freestyle canoeing approx. 1.5 m**
- **flow velocity - approx. 2 m/s, with some sections of greater speed**
- **material - concrete - roughness coefficient safe for boats and people**
- **movable obstacles, platforms, etc. - to satisfy multiple-user requirements.**



## Whitewater courses – multifunctional world's canoe slalom venues

Name, Location, Year	Use	Length (m)	Drop (m)	Slope (%)	Flow rate (m <sup>3</sup> /s)
Eiskanal, Augsburg, Germany, 1972	Competition, Training	305	3.2	1.3	10
Parc Olímpic del Segre, Catalonia, Spain, 1990	Competition	300	6.5	2.2	12-17.5
	Beginners	130	1.5	1.2	3-10
Penrith Whitewater Stadium, Sydney, Australia, 1999	Competition, Training, Reception	320	5.5	1.7	14
Hellinikon Olympic Canoe/Kayak Slalom Centre, Athens, Greece, 2004	Competition, Training	270	6	1-2	17.5
U.S. National Whitewater Center, North Carolina, USA, 2006	Competition, Training	300	6.4	2.1	15
	Recreation	550		1.2	19
Shunyi Olympic Rowing-Canoeing Park, Benjin, China 2007	Competition, Training	300	6.3	2.1	17.5
Lee Valley White Water Centre, London, UK, 2010	Comepetition	300	5.5	1.8	13
	Beginners	160	1.6		10.5
Deodoro, Rio de Janeiro, Brazil, 2015	Competition	250	4.5	1.8	12
	Training	200	2	1	10.5



## Whitewater courses – Munich (1972) – Eiskanal in Augsburg

- The canoe course was designed as copy of natural watercourse. The 300-meter long venue was built of concrete simulating rocks and boulders fixed to irregular bed and walls of the channel.



[Source: Wikipedia](#)



## Whitewater courses – London (2012) – Lee Valley White Water Centre

- The course was designed as artificial canal with movable obstacles . The plastic blocks were attached to the channel bed to create whitewater effects.

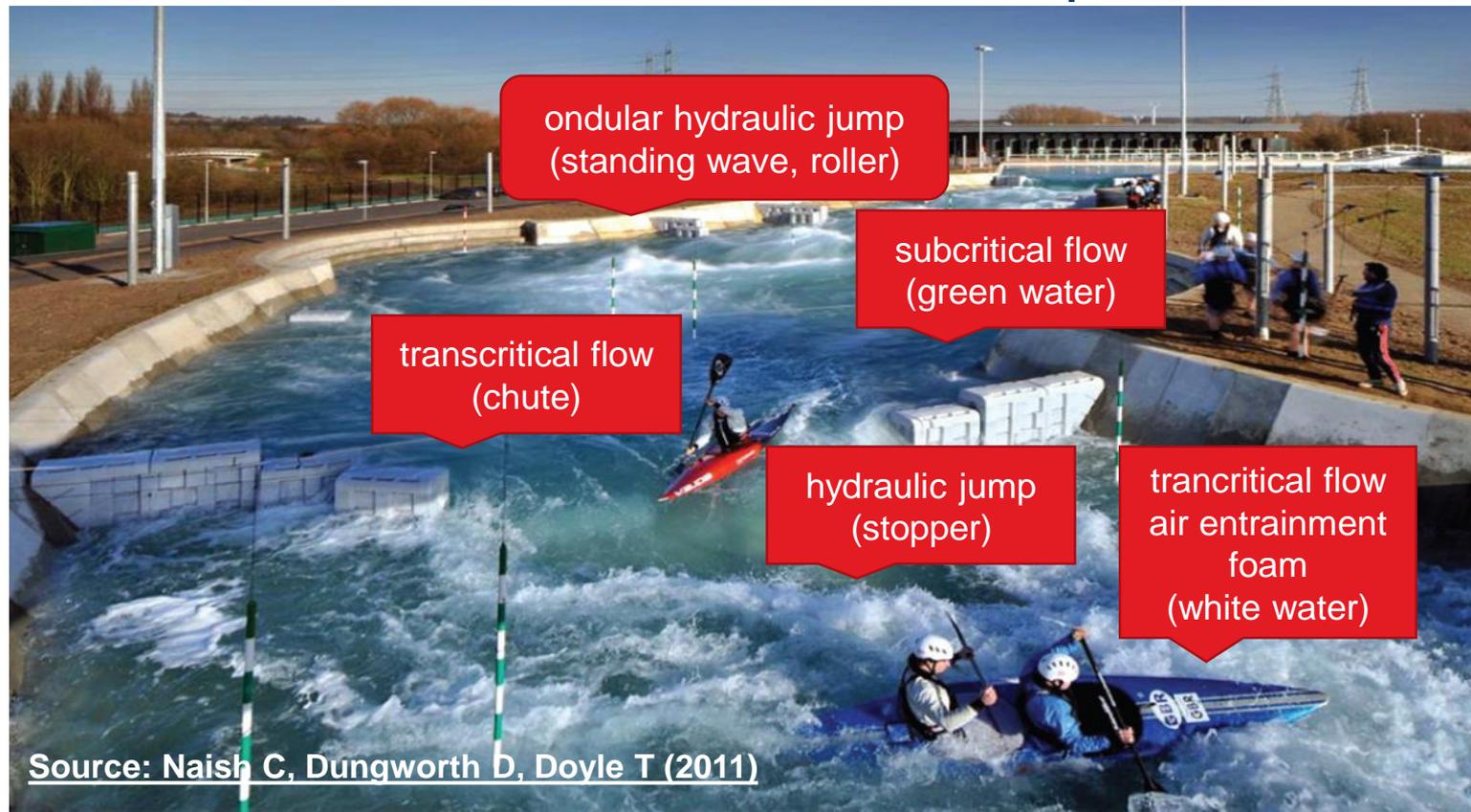


Source: Naish C, Dungworth D, Doyle T (2011)



## Whitewater courses – hydraulic aspects of designing

- The key strategy for successful slalom course design is the proper recognition of local hydraulic effects combined with an exact estimation of water depths and flow velocities.

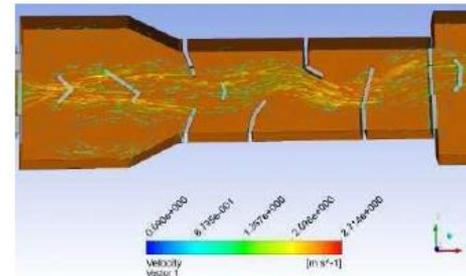
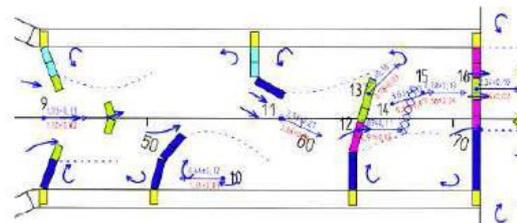


Source: Naish C, Dungworth D, Doyle T (2011)



## Whitewater courses – laboratory and mathematical modeling

- The current main approach for designing and testing whitewater courses is to use a combination of the Froude scale hydraulic model and prior experience, complemented by verification with CFD 3D models.





## Whitewater courses – laboratory and mathematical modeling

- In our opinion SWE model seems to be precise enough to represent the hydraulic behavior of rapidly varied flow at the concept step of whitewater course design.
- SWE were solved for rapidly varied flow using Finite Volume Method (FVM).

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \mathbf{S} = 0$$

$$\mathbf{U} = \begin{pmatrix} h \\ uh \\ vh \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ -gh(S_{ox} - S_{fx}) \\ -gh(S_{oy} - S_{fy}) \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} uh \\ u^2h + 0.5gh^2 \\ uvh \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} vh \\ uvh \\ v^2h + 0.5gh^2 \end{pmatrix}$$



## Whitewater courses – laboratory and mathematical modeling

- 2D modeling of canoeing whitewater course in Drzewica, Poland.



Source: Szydłowski M (2016)



## Whitewater courses – laboratory and mathematical modeling

- Good quality of results but no measurements, no verification.



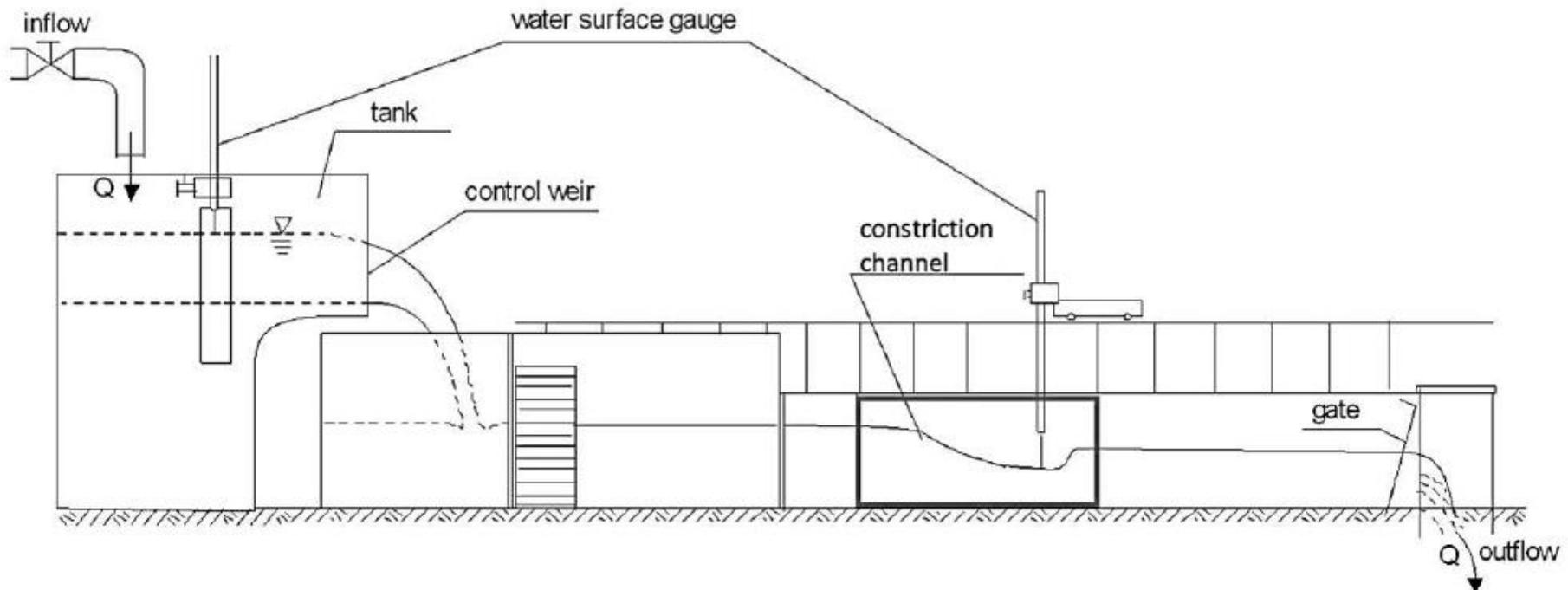
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## Whitewater courses – laboratory and mathematical modeling

- Scheme of the laboratory open channel set-up.

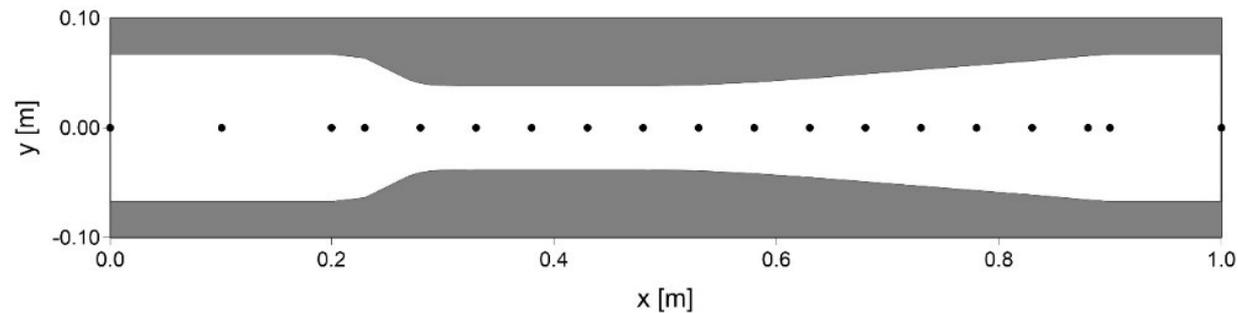
$$Q = \text{const.} = 0.0172 \text{ m}^3/\text{s}$$





## Whitewater courses – laboratory and mathematical modeling

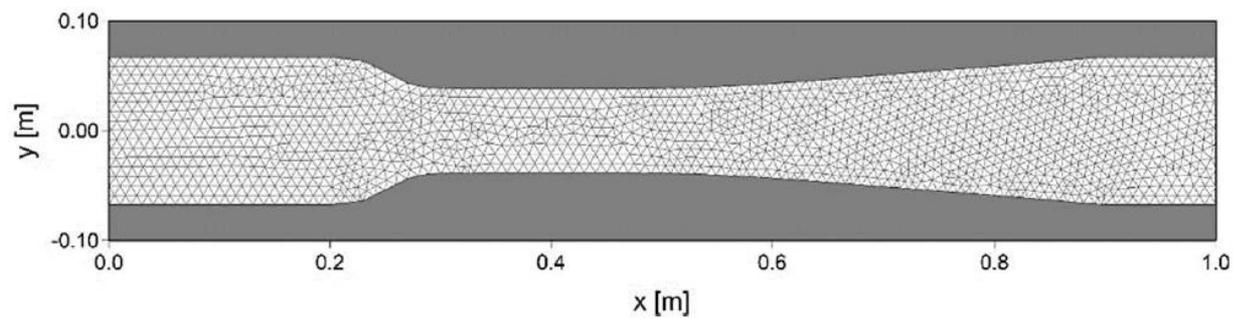
- View and geometry of the laboratory channel with a constriction.





## Whitewater courses – laboratory and mathematical modeling

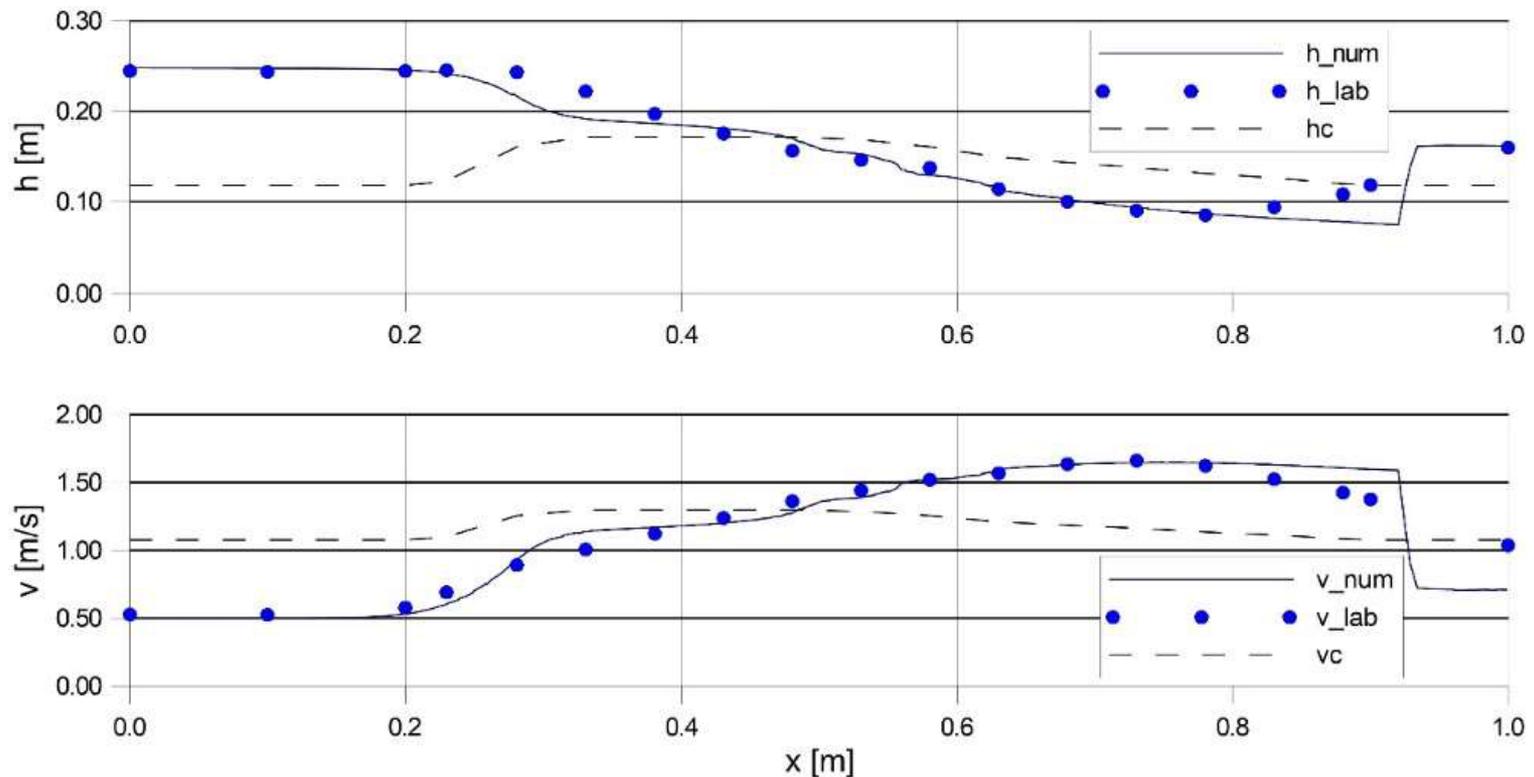
- Laboratory experiment and numerical mesh.





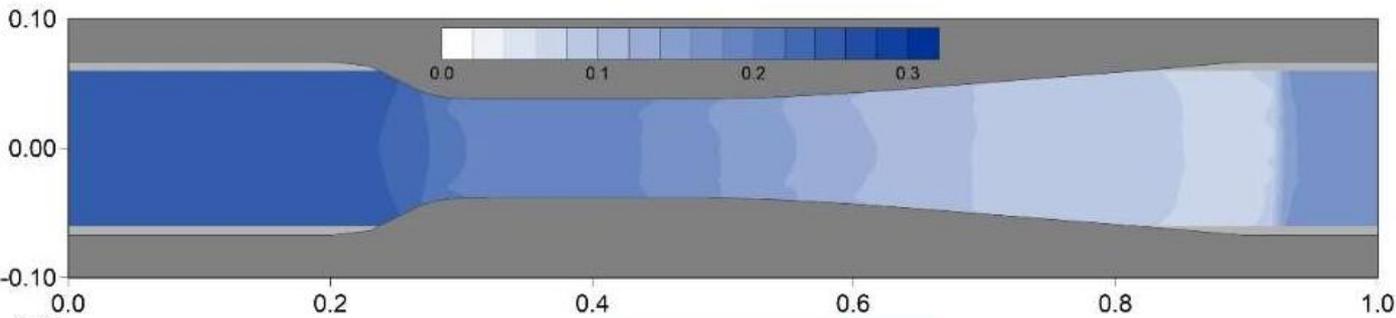
## Whitewater courses – laboratory and mathematical modeling

### Results – measurements and calculations.

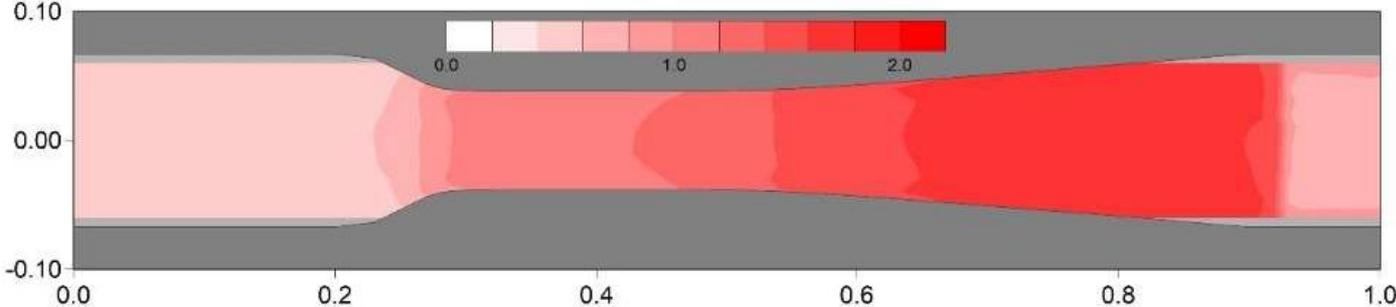




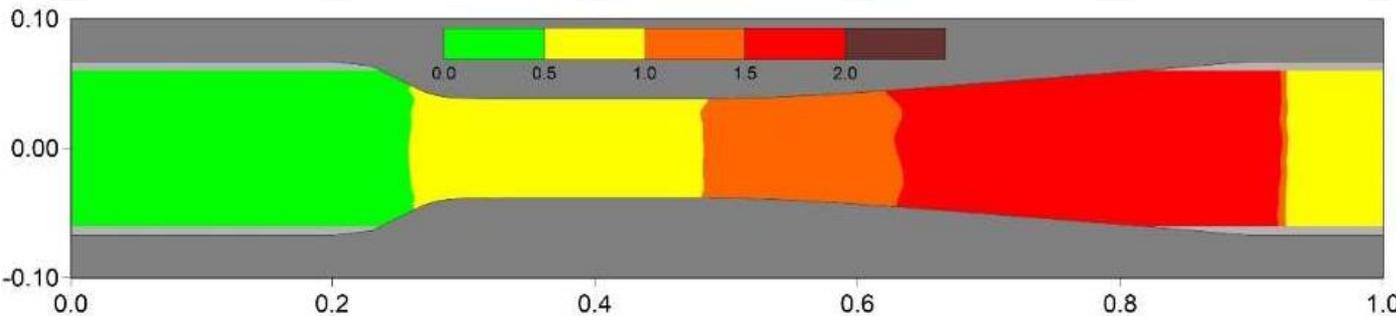
## Whitewater courses – 2D numerical simulation



■ Depth (m)



■ Velocity (m/s)



■ Froude number (-)



## **CONCLUSIONS**

**The main hydraulic parameters of water flow in the channel constriction was properly approximated.**

**The inner structure of the local whitewater flow phenomena is lost due to the vertical averaging of flow parameters.**

**Shallow water equations can be successfully used for modelling the flow in whitewater courses.**

**2D numerical simulation can be useful only at the conceptual stage of the design process.**



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**HISTORY IS WISDOM  
FUTURE IS CHALLENGE**