Theoretical Analysis of the Reduction of Pressure Wave Velocity by Internal Circular Tubes

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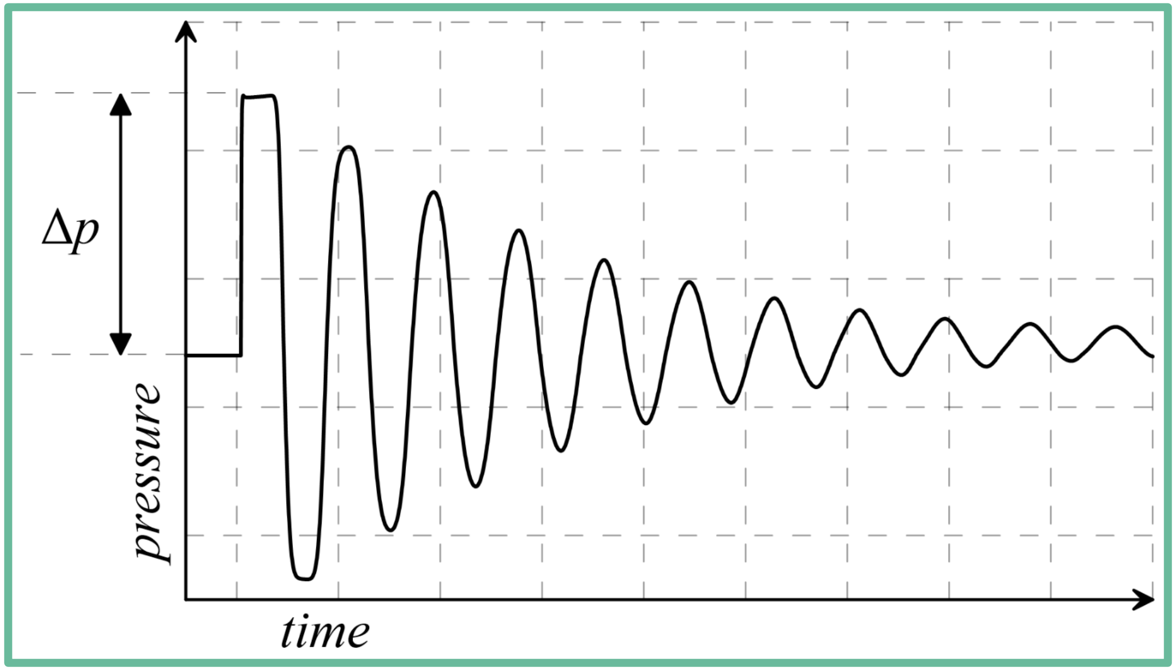
The water hammer phenomenon



Joukowsky equation

$$\Delta p = \rho c v_0$$

where:

Δp – pressure increase [Pa],

ρ – water density [kg·m^{-3}],

c – pressure wave velocity [m·s^{-1}],

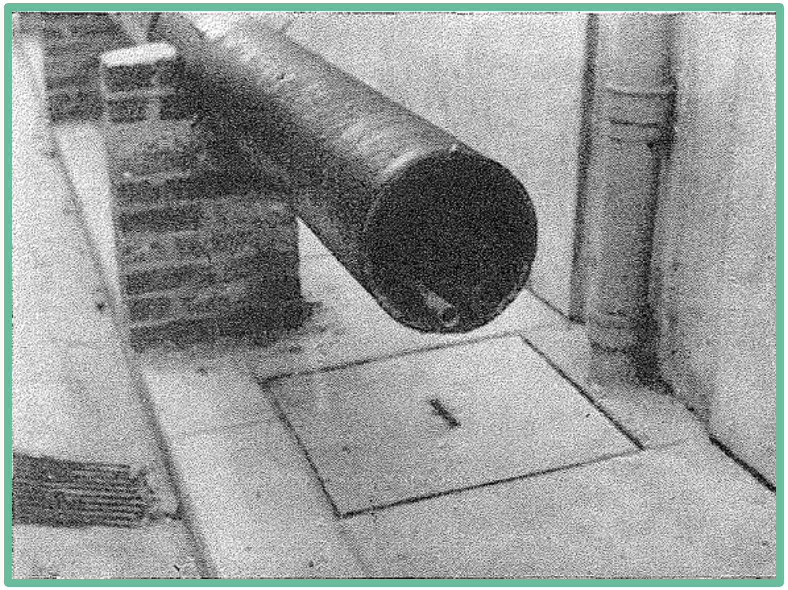
Δv_0 – initial water flow velocity [m·s^{-1}].

The water hammer protection devices

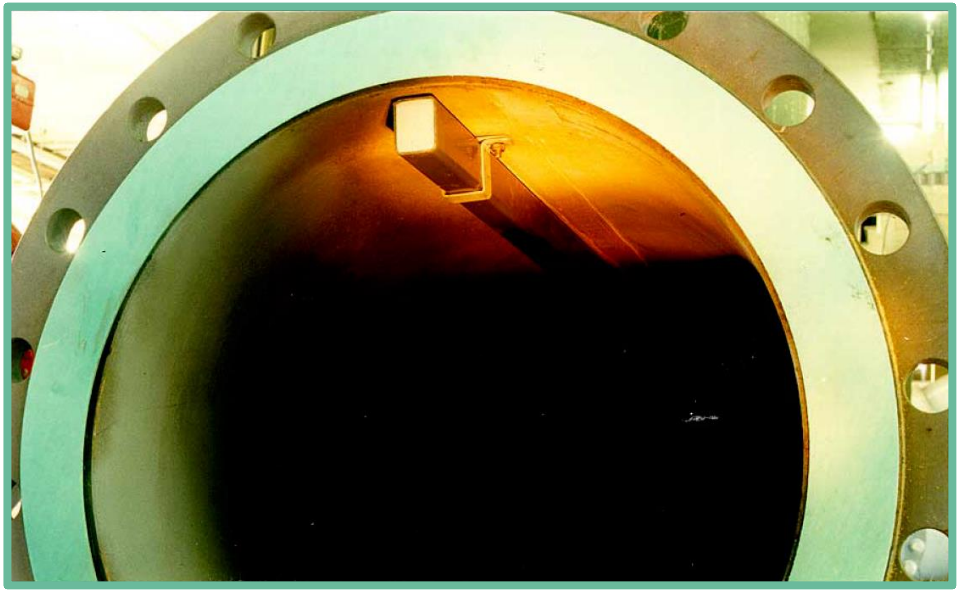
safety valve

surge tanks

Previous work



Remenieras (1952)



Tijsseling et al. (1999)

Water hammer equations

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial H}{\partial x} + \frac{f}{2D}v|v| = 0$$

momentum equation

$$\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial x} + \frac{c^2}{g}\frac{\partial v}{\partial x} = 0$$

continuity equation

where:

x – space co-ordinate [m],

t – time [s],

c – pressure wave velocity [$m \cdot s^{-1}$],

Δv – water flow velocity [$m \cdot s^{-1}$],

f – Darcy-Weisbach friction factor [$m \cdot s^{-1}$],

D – inner diameter of the pipeline [m],

g – gravitational acceleration [$m \cdot s^{-2}$].

Water hammer equations

$$\Delta p = \rho c v_0$$ Joukowsky equation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial H}{\partial x} + \frac{f}{2D} v|v| = 0$$ momentum equation

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where:

c – pressure wave velocity [$m \cdot s^{-1}$].

Pressure wave velocity

$$c = \frac{\sqrt{\frac{K}{\rho}}}{\sqrt{1+\frac{KD}{Ee}}}$$

Korteweg-Joukowsky equation

where:

K – water compressibility [Pa],

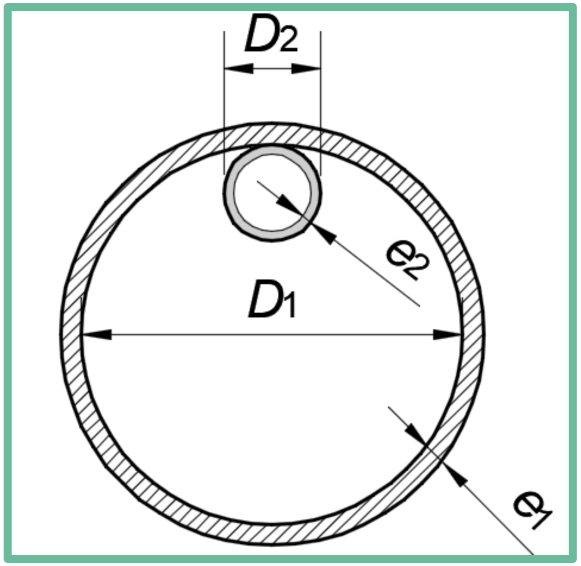
ρ – water density [kg·m^{-3}],

D – internal diameter of the pipeline [m],

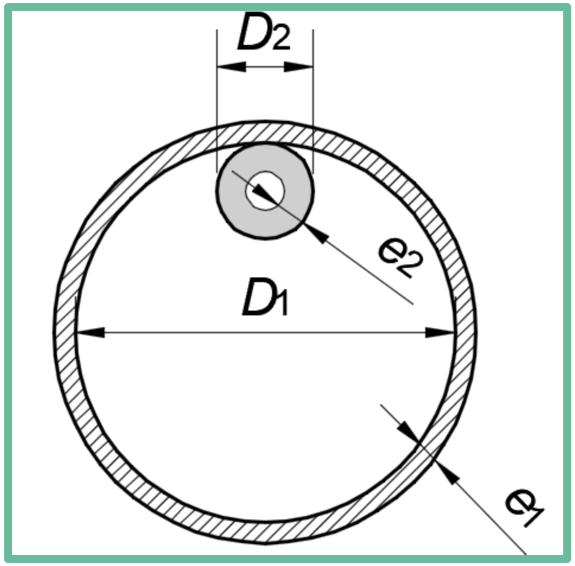
E – Young's modulus of the pipeline wall [Pa],

e – pipeline wall thickness [m].

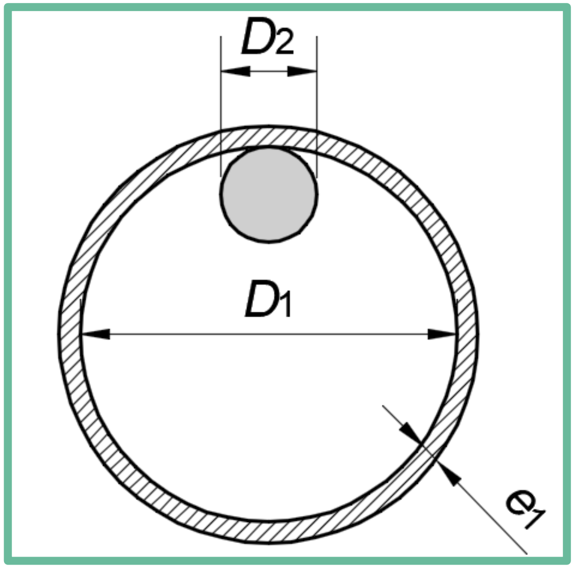
Different types of tubes



thin-walled
tube



thick-walled
tube



solid cylindrical
tube

General assumptions

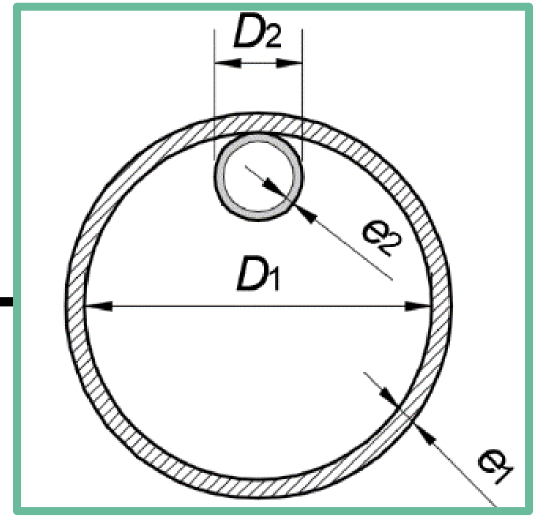
- Perfect elastic behavior of the liquid, pipeline and tube.
- The pipeline is thin-walled and, therefore, only hoop stress was taken into account.
- During steady flow, the liquid pressure in the pipeline is the same as the pressure of the gas in the tube.
- The air pressure in the tube is much smaller than Young's modulus of the tube's material.

Conservation of energy

$$E_k = E_l + E_p + E_t$$

change of the kinetic energy of liquid stream	=	increase of the elastic energy of liquid stream	+	increase of the elastic energy of the pipeline	+	increase of the elastic energy of the tube

Type of the pipeline system	Pressure wave velocity formula
Pipeline with inserted **thin-walled tube**	$c_n = \sqrt{\dfrac{\frac{K}{\rho}}{1 + \frac{A_1}{A}\frac{KD_1}{E_1 e_1} + \frac{A_2}{A}\frac{KD_2}{E_2 e_2}}}$
Pipeline with inserted **thick-walled tube**	$c_k = \sqrt{\dfrac{\frac{K}{\rho}}{1 + \frac{A_1}{A}\frac{KD_1}{E_1 e_1} + \frac{A_2}{A}\frac{K}{E_2}\left(\frac{D_2}{e_2} - 1\right)}}$
Pipeline with inserted **solid circular tube**	$c_c = \sqrt{\dfrac{\frac{K}{\rho}}{1 + \frac{A_1}{A}\frac{KD_1}{E_1 e_1} + \frac{A_2}{A}\frac{K}{E_2}}}$



where:
E_1 – Young's mod. of the pipeline wall [Pa],
E_2 – Young's mod. of the tube wall [Pa],
A – cross-section area of the stream [m^2],
A_1 – cross-section area of the pipeline [m^2],
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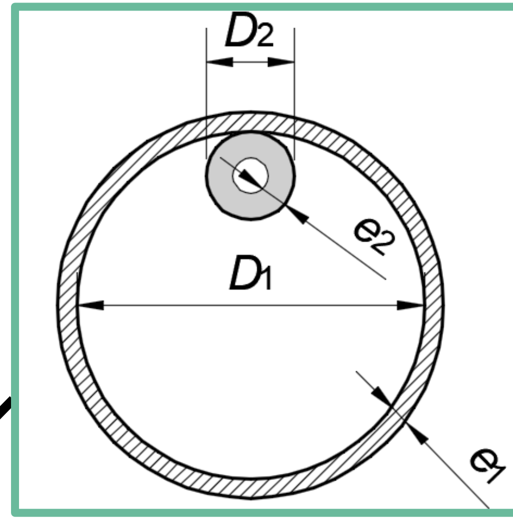
Pipeline with inserted **solid circular tube**	$c_c = \sqrt{\dfrac{\dfrac{K}{\rho}}{1 + \dfrac{A_1}{A}\dfrac{KD_1}{E_1 e_1} + \dfrac{A_2}{A}\dfrac{K}{E_2}}}$



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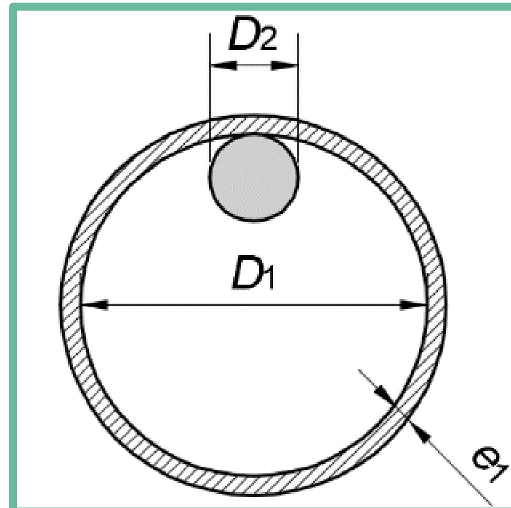

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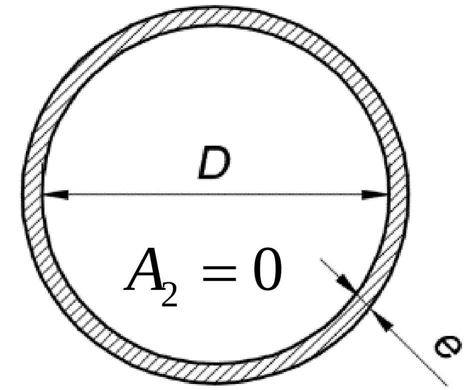
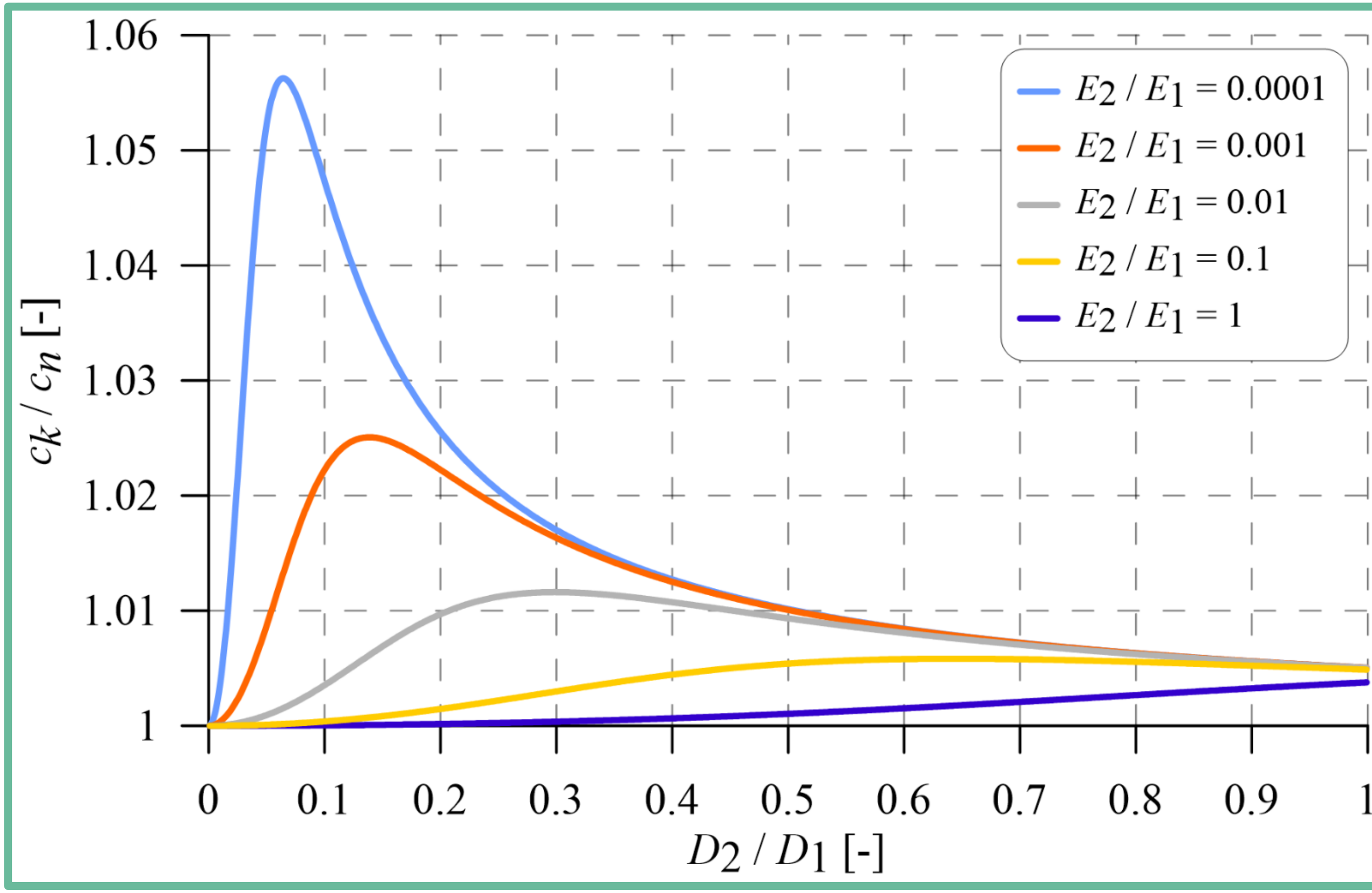


$$c = \frac{\sqrt{\dfrac{K}{\rho}}}{\sqrt{1+\dfrac{KD}{Ee}}}$$

Comparison of pressure wave velocity values

pipeline with thin-walled tube

pipeline with thick-walled tube



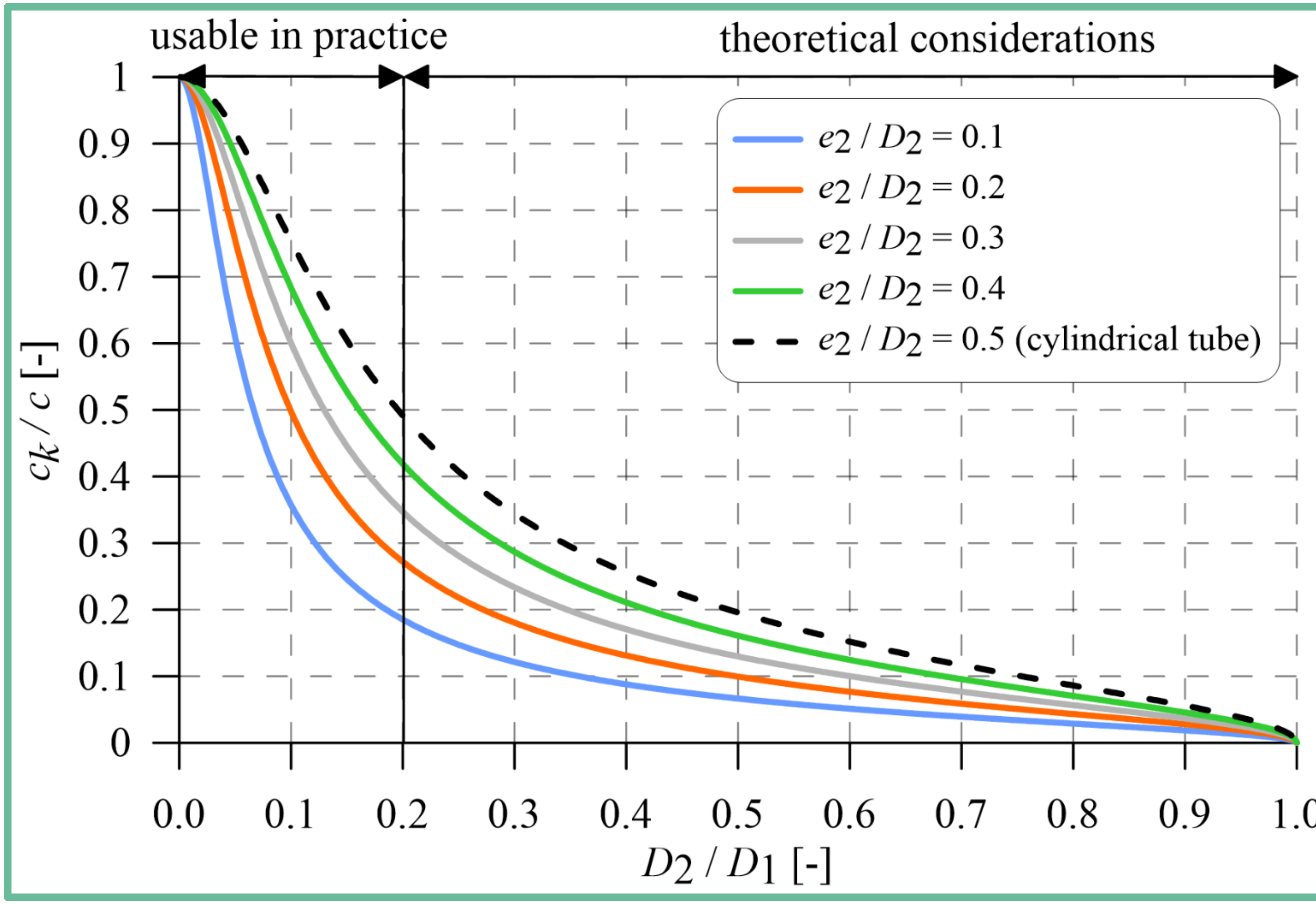
outer diameter of the tube

inner diameter of the pipeline

Comparison of pressure wave velocity values

pipeline without the tube

pipeline with thick-walled tube



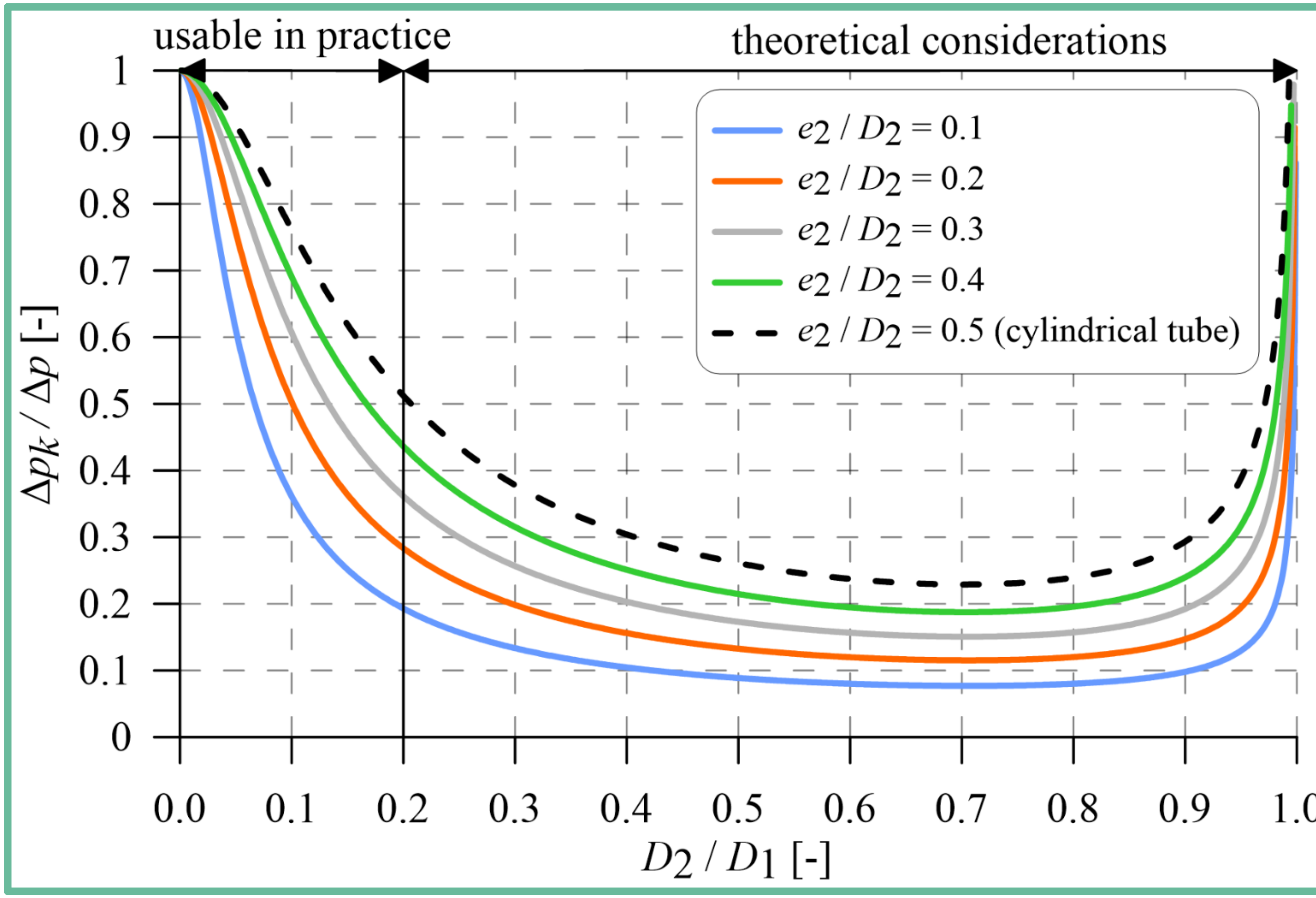
outer diameter of the tube

inner diameter of the pipeline

Comparison of pressure increase values

pipeline without the tube

pipeline with thick-walled tube



outer diameter of the tube

inner diameter of the pipeline

Conclusions and summary

- In the analysis three types of tubes were distinguished: thin-walled tube, thick-walled tube and solid cylindrical tube. For each of these cases, using the work-energy principle, a formula for calculating the pressure wave velocity was derived.

- The values calculated for thin-walled tubes are smaller than those calculated for thick-walled tubes. However, for the assumed range of calculations, these differences are negligibly small.

Conclusions and summary

- The formula for the pressure wave velocity in a pipeline with a thick-walled tube has a wider application because by substituting $e_2 = D_2/2$, it transforms into a formula for the velocity of pressure wave with a solid cylindrical tube.

- It was shown that insertion of a tube with low bulk elastic modulus, may have a damping effect on the water hammer phenomenon, i.e., it reduces the pressure wave velocity and maximum pressure increase. The damping properties of the tubes are higher when the Young's modulus and wall thickness are lower.

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Results of the experimental tests